Summary. Ecological modelers produce models with more and more details, leading to dynamical systems involving lots of variables. This chapter presents a set of methods which aim to extract from these complex models some submodels containing the same information but which are more tractable from the mathematical point of view. This “aggregation” of variables is based on time scales separation methods. The first part of the chapter is devoted to the presentation of mathematical aggregation methods for ODE’s, discrete models, PDE’s and DDE’s. The second part presents several applications in population and community dynamics.

5.1 Introduction

Ecology aims to understand the relations between living organisms and their environment. This environment constitutes a set of physical, chemical and biological constraints acting at the individual level. In order to deal with the complexity of an ecosystem, ecology has been developed on the basis of a wide range of knowledge starting from the molecular level (molecular ecology) to the ecosystem level. One of the current aims of ecological modelling is to use the mathematical formalism for integrating all this knowledge.
On the other hand, mathematical ecology provided a large amount of rather simple models involving a small number of state variables and parameters. The time continuous Lotka–Volterra models, published in the beginning of the twentieth century \cite{59,60,94} as well as the discrete host–parasite Nicholson–Bailey models \cite{69,70} are classic examples and can be found in many biomathematical textbooks as the book by Edelstein-Keshet \cite{48} and the book by Murray \cite{66} in which many other examples and references are given. In such population dynamics models, the state variables are often chosen as the population densities and the model is a set of nonlinear coupled ordinary differential equations (ODE’s) or discrete equations. The models describe the time variation of the interacting populations. Of course, mathematical ecologists proposed also more realistic models taking account of some populations structures (space, age, physiology, etc.). Mathematical methods have been developed to deal with these structured population models, but which may fail to get robust results for high dimensional systems.

During the last decades, supported by the fast development of computers, a new generation of ecological models has appeared. Nowadays, lots of ecological models consider more and more details. Lots of populations are involved in a community and in food webs. Furthermore, each population is not homogeneous in the sense that all individuals are identical but each individual has changing properties (physiology, metabolism, behaviour), according to its environment. For instance, functional ecology considers functional groups corresponding to the different functions of living organisms in the ecosystems. It follows that lots of models consider populations structured in subgroups.

Incorporating more details in models is necessary to advance toward a more realistic description of ecological systems and to understand how living organisms respond to the forcing imposed by their environment changes. The drawback of a detailed description of systems is the fact that models become more complex, involving an increasing number of variables and parameters. A mathematical study with general and robust results is then difficult to perform. For this reason, it is important to find which details are really relevant and must be incorporated in a model. An important goal of ecological modelling should thus be to describe tractable models.

In the context of terrestrial ecology, if we consider a forest dynamics for instance, we can just consider the total forest surface or how trees are distributed among species or globally. But the dynamics of these variables or indicators depend on the individuals properties of the trees (height, weight, basal area, metabolism, etc.). Should we take into account all the details? Is there a trade-off between the amount of details to be integrated and the relative simplicity required for understanding forest dynamics? In this case, the surface or the spatial structure indicators are global variables that we call macro or aggregated variables. These variables actually depend on the individuals descriptors, which we call the micro-variables.

The same approaches are considered in marine ecology. The simplest way should be to consider the concentrations of mineral matter, primary producers, zooplankton, top-predators and microbial loop with bacteria and detritus