8 Multivariable PID Control for Synchronization

Chapters 7 presents the new design methods for MIMO PID control in the framework of LMI and optimal control. In this chapter, some of these design methods are also applied to solve a practical problem — to achieve fast master-slave synchronization of Lur’e systems with multivariable PD/PID control.

8.1 Introduction

Since the seminal work of Pecora and Carroll [177], the topic of chaos synchronization has attracted great interest in both theoretical studies [178, 179, 180, 181, 182, 183] and practical applications [184, 185, 186, 187, 188]. Synchronization phenomena in nonidentical systems or the coupled systems with different order have been investigated [189]. A number of master-slave synchronization schemes for Lur’e systems have been proposed [183, 190, 182, 191]. Wen et al. [181] studied robust synchronization of chaotic systems under output feedback control with multiple random delays. Some linear-state-feedback synchronization control methods [192, 193, 194, 195] are reported. For example, in the reference [194], it is proved that global asymptotic synchronization can be attained via a linear output error feedback approach when the feedback gain chosen as a function of a free parameter is large enough. Yassen [196] investigated chaos synchronization by using adaptive control. Chaos synchronization has also been addressed using observers with linear output feedback [197], PI observers [190, 198, 199] and nonlinear observers [200, 201, 202]. Femat et al. proposed a Laplace domain controller and its applications to design PII² controller [203, 204]. Their results enable one to observe the different synchronization phenomena of chaotic systems with different order and model [180]. Jiang and Zheng [192] proposed a linear-state-feedback synchronization criterion based on LMI. To our best knowledge, no work is reported in the literature on chaos synchronization via full PID control. This may result from the fact in PID control studies that many prevalent PID controller design methods were established on basis of frequency response methods. The intrinsic characteristic of synchronization implies that the state-space approach is preferable for serving our purpose.
Hua and Guan [199] transformed chaotic systems with PI controller into an augmented proportional control system, as reported earlier by Zheng et al. [79], and proposed a synchronization criterion using the LMI technique. However, their methodology is not applicable to chaotic systems with PD or PID controller. It is well known in the area of PID control that the integral control is mainly employed to improve the steady state tracking accuracy while the derivative control to enhance stability and speed the system response [2]. Obviously, it is desirable to enhance stability and speed synchronization response as concerning the chaos synchronization. This implies that the derivative control is desirable to increase synchronization speed for Lur’e systems. If there had existed appropriate design methods of PID controller, PID control for synchronization should have prevailed.

The objective of this chapter is to propose a multivariable PD/PID controller design to achieve fast master-slave synchronization of Lur’e systems [205]. Due to the fact that measuring all the state variables of a system is inconvenient or even impossible in many practical situation [206], output feedback control is considered. The free-weighting matrix approach [109, 110, 111, 207] and the S-procedure [89] are employed to establish the synchronization strategy. It is shown that our corollary covers the existing result in the case of proportional control alone. Numerical results demonstrate the improvement of speeding synchronization response with the aid of the derivative action, as compared to the results of the same chaotic system based on PI control [199].

### 8.2 Problem Formulation

Consider a general master-slave type of coupled Lur’e systems with PD/PID controller:

\[
\mathcal{M} : \begin{cases}
    \dot{x}(t) &= Ax(t) + B\sigma(C^T x(t)) \\
    y(t) &= Hx(t)
\end{cases}
\]

\[
\mathcal{S} : \begin{cases}
    \dot{x}(t) &= A\hat{x}(t) + B\sigma(C^T \hat{x}(t)) + u(t) \\
    \hat{y}(t) &= H\hat{x}(t)
\end{cases}
\]

\[
\mathcal{C} : \begin{cases}
    u(t) &= K_p(y(t) - \hat{y}(t)) + K_d(\dot{y}(t) - \dot{\hat{y}}(t)) \\
    u(t) &= K_p(y(t) - \hat{y}(t)) + K_i \int_0^t (y(\theta) - \hat{y}(\theta)) d\theta + K_d(\dot{y}(t) - \dot{\hat{y}}(t))
\end{cases}
\]

with master system \( \mathcal{M} \), slave system \( \mathcal{S} \) and controller \( \mathcal{C} \). The master and slave systems are Lur’e systems with control input \( u \in \mathbb{R}^n \), state vectors \( x, \hat{x} \in \mathbb{R}^n \), and matrices \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times nh}, C \in \mathbb{R}^{n \times nh} \). The matrix \( H \in \mathbb{R}^{l \times n} \) implies use of the output feedback and the outputs of subsystems are \( y, \hat{y} \in \mathbb{R}^l \), respectively. The nonlinearity \( \sigma(\cdot) = [\sigma_1, \sigma_2, \cdots, \sigma_{nh}]^T \) satisfies a sector condition with \( \sigma_j(\cdot), j = 1, 2, \cdots, nh \), belonging to sectors \( [0, k_j] \), i.e.,

\[
\sigma_j(\xi)(\sigma_j(\xi) - k_j \xi) \leq 0, \forall \xi, \text{ for } j = 1, 2, \cdots, nh.
\]