Chapter 18:
Digital-Analog and Analog-Digital Converters

To display or process a voltage digitally, the analog signal must be translated into numeric form. This task is performed by an analog-to-digital converter (A/D converter, or ADC). The resultant number $Z$ will generally be proportional to the input voltage $V_i$:

$$Z = \frac{V_i}{V_{\text{LSB}}}$$

where $V_{\text{LSB}}$ is the voltage unit for the least significant bit; that is, the voltage for $Z = 1$.

To convert a number back into a voltage, a digital-to-analog converter (D/A converter, or DAC) is used, whose output voltage is proportional to the numeric input; that is,

$$V_o = V_{\text{LSB}} \cdot Z$$

18.1 Sampling Theorem

A continuous input signal can be converted into a series of discrete values by using a sample-and-hold circuit for sampling the signal at equidistant instants $t_\mu = \mu T_s$, where $f_s = 1/T_s$ is the sampling rate. It is obvious from Fig. 18.1 that a staircase function arises, and that the approximation to the continuous input function is better the higher the sampling rate. However, as circuit complexity increases markedly with higher sampling rates, it is essential to keep the latter as low as possible. The question is now: What is the lowest sampling rate at which the original signal can still be reconstructed error-free; that is, without loss of information. This theoretical limit is defined by the sampling theorem (the Nyquist criterion), which we shall now discuss.

In order to obtain a simpler mathematical description, the staircase function shown in Fig. 18.1 is replaced by a series of Dirac impulse functions, as illustrated in Fig. 18.2:

$$\tilde{V}_i(t) = \sum_{\mu=0}^{\infty} V_i(t_\mu) T_s \delta(t - t_\mu)$$  \hspace{1cm} (18.1)

Their impulse area $V_i(t_\mu) \cdot T_s$ is represented by an arrow. The arrow must not be mistaken for the height of the impulse, as a Dirac function is, by definition, an impulse with infinite height but zero width, although its area has a finite value. This area is often misleadingly

Fig. 18.1. Example of an input signal $V_i(t)$ and sampled values $V_i(t_\mu)$

Fig. 18.2. Representation of the input signal by a Dirac impulse sequence

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known as the impulse amplitude. The characteristics of the impulse function are shown by Fig. 18.3, where the Dirac impulse function is approximated by a rectangular pulse $r_\varepsilon$; the limit of the approximation is

$$V_i(t_\mu)T_\varepsilon \delta(t - t_\mu) = \lim_{\varepsilon \to 0} V_i(t_\mu)r_\varepsilon(t - t_\mu)$$

(18.2)

To examine the information contained in the impulse function sequence represented by (18.1), we consider its spectrum. By applying the Fourier transformation to (18.1), we obtain

$$\tilde{X}(jf) = T_\varepsilon \sum_{\mu=0}^{\infty} V_i(\mu T_\varepsilon) e^{-2\pi j \mu f / f_s}$$

(18.3)

It can be seen that this spectrum is a periodic function, the period being identical to the sampling frequency $f_s$. When this periodic function is Fourier analyzed, it can be shown that the spectrum $|\tilde{X}(jf)|$ is, for $-\frac{1}{2} f_s \leq f \leq \frac{1}{2} f_s$, identical to the spectrum $|\tilde{X}(jf)|$ of the original waveform. Thus it still contains all of the information, although only a few values of the function were sampled.

There is only one restriction, and this is explained with the help of Fig. 18.4. The original spectrum reappears unchanged only if the sampling rate is chosen such that consecutive bands do not overlap. According to Fig. 18.4, this is the case for

$$f_s > 2 f_{\text{max}}$$

(18.4)

this condition being known as the sampling theorem.