Analysis of Closed-Loop Performance and Frequency-Domain Design of Compensating Filters for Sliding Mode Control Systems

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1 Introduction

It is known that the presence of parasitic dynamics in a sliding mode (SM) system causes high-frequency vibrations (oscillations) or chattering [1]-[3]. There are a number of papers devoted to chattering analysis and reduction [4]-[10]. Chattering has been viewed as the only manifestation of the parasitic dynamics presence in a SM system. The averaged motions in the SM system have always been considered the same as the motions in the so-called reduced-order model [11]. The reduced-order model is obtained from the original equations of the system under the assumption of the ideal SM in the system. Under this assumption, the averaged control in the reduced-order model becomes the equivalent control [11]. This approach is well known and a few techniques are developed in details. However, the practice of the SM control systems design shows that the real SM system cannot ensure ideal disturbance rejection. Therefore, if the difference between the real SM and the ideal SM is attributed to the presence of parasitic dynamics in the former then the parasitic dynamics must affect the averaged motions. Yet, the effect of the parasitic dynamics on the closed-loop performance can be discovered only if a non-reduced-order model of averaged motions is used. Besides improving the accuracy, the non-reduced-order model would provide the capability of accounting for the effects of non-ideal disturbance rejection and non-ideal input tracking. The development of the non-reduced order model becomes feasible owing to the locus of a perturbed relay system (LPRS) method [12] that involves the concept of the so-called equivalent gain of the relay, which describes the propagation of the averaged motions through the system with self-excited oscillations. Further, the problems of designing a predetermined frequency of chattering and of the closed-loop performance enhancement may be posed. The solution of those two problems may have a significant practical impact, as it is chattering that prevents the SM principle from a wider practical use, and it is performance deterioration not accounted for during the design that creates the situation of “higher expectations” from the SM principle.

In the present chapter, analysis of chattering and of the closed-loop performance is carried out in the frequency domain via the LPRS method. Further,
it is proposed that the effects of the non-ideal closed-loop performance can be mitigated via the introduction of a linear compensator. The methodology of the compensating filter design is given.

The chapter is organized as follows. At first, the approach to the analysis of averaged motions in a SM control system is outlined, and the concept of the equivalent gain of the relay with respect to the averaged signals is introduced. After that the basics of the LPRS method are given. In the following section, the non-reduced-order model of the averaged motions is presented. Then, a motivating example illustrating the problem is considered. In the following section, the compensation mechanism is analyzed. Finally, an illustrative example of the compensating filter design is given.

2 Averaged Motions in a Sliding Mode System

It is known that the SM control is essentially a relay control with respect to the sliding variable. Therefore, the SM system can be analyzed as a relay system. If no parasitic dynamics and switching imperfections were present in the system the ideal SM would occur. It would feature infinite frequency of switching of the relay and infinitely small amplitude of the oscillations at the system output. However, the inevitable presence of parasitic dynamics (in the form of the actuator and sensor dynamics) in series with the plant and switching imperfections of the relay result in the finite frequency of switching and finite amplitude of the oscillations, which is usually referred to as chattering. Chattering was the subject of analysis of a number of publications [1]-[4], [11] and was analyzed as a periodic motion caused by the presence of fast actuator dynamics. To obtain the model of averaged motions in a SM control system that has parasitic dynamics and switching imperfections, let us analyze the relay feedback system under a constant load (disturbance) or a constant input applied (Fig.1). At first obtain the model that relates the averaged values of the variables when a constant input is applied to the system. Later we can extend the results obtained for the constant input (and constant averaged values) to the case of slow inputs.

Let the SM system be described by the following equations that would comprise both: the principal dynamics and the parasitic dynamics, which are assumed to be type 0 servo system (not having integrators):

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\] (1)