The Local and Global Price of Anarchy of Graphical Games

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Abstract. This paper initiates a study of connections between local and global properties of graphical games. Specifically, we introduce a concept of local price of anarchy that quantifies how well subsets of agents respond to their environments. We then show several methods of bounding the global price of anarchy of a game in terms of the local price of anarchy. All our bounds are essentially tight.

Keywords: Graphical games, price of anarchy, local global properties.

1 Introduction

The model of graphical games [10], is a recent representation method of games in which the dependencies among the agents are represented by a graph. In a graphical game, each agent is identified by a vertex, and its utility is determined solely by its own action and the actions of its graph neighbors. Note that every game can be represented by a graphical game with a complete graph. Yet, often, a much more succinct representation is possible. While the original motivation of defining graphical games was computational, we believe that an important property of the model is that it enables an investigation of many natural structural properties of games.

In this work we investigate connections between local and global properties of graphical games. Specifically, we study the Price of Anarchy (PoA) which is the ratio between the welfare of a worst Nash equilibrium and the optimal possible welfare [11].

We introduce a novel notion of a local price of anarchy which quantifies how well subsets of agents respond to their environments. We then study the relations between this local measure and the global price of anarchy of the game. We provide several methods of bounding the global price of anarchy in terms of the local price of anarchy, and demonstrate the tightness of these methods.

One possible interpretation of our results is as follows: if a decentralized system is comprised of smaller, well behaved units, with small overlap between them,
then the whole system behaves well. This holds independently of the size of the small units, and even when the small units only behave well on average. This phenomenon may have implications, for example, on organizational theory. From a computational perspective, the price of anarchy of large games is likely to be extremely hard to compute. However, computing the local price of anarchy of small units is relatively easy since they correspond to much smaller games. Once these are computed, our methods can be invoked to bound the price of anarchy of the overall game.

1.1 Related Work

The model of graphical games was introduced in [10]. The original motivation for the model was computational as it permitted a succinct representation of many games of interest. Moreover, for certain graph families, there are properties that can be computed efficiently. For example, although computing a Nash equilibrium is usually a hard task [5], it can be computed efficiently for graphical games on graphs with maximum degree 2 [6]. Rather surprisingly, the proofs of the hardness of computing Nash equilibria of normal form games are conducted via reductions to graphical games [5].

[8] investigates the structure of equilibria of graphical games under some symmetry assumptions on the utility of the agents. It shows that in these games, there always exists a pure strategy equilibrium. For such games of incomplete information, [8] shows that there is a monotone relationship between the degree of a vertex and its payoff, and investigates further the connections between the level of information the game possesses and the monotonicity of the players’ degree in equilibria. Several works coauthored by Michael Kearns explore economic and game theoretic properties which are related to structure (e.g. [9]). The questions addressed in these works are very different from the ones we address here.


[12] investigates deductions that can be made on global properties of graphs after examining only local neighborhoods. It shows that for any graph $G$, where $V(G) = n$, and a function $f : V \to \mathbb{R}^+ \cup \{0\}$, if the local average of $f$ over every ball of positive radius less or equal to $r$ in $G$ is greater or equal to $\alpha$, the global average of $f$ is at least $\frac{n}{r} \cdot \frac{9}{\log(r)}$. [12] also demonstrates the tightness of this bound.

In this work we make an extensive use of graph covers, but we do not introduce a method for finding them. Algorithms that find good covers can be found, for example, in [3] and [13]. Due to the game theoretic nature of our setup, these algorithms cannot be applied to it directly.

In general, the field of property testing in computer science examines the connections between local and global properties of combinatorial objects (see,

\footnote{The price of stability is the ratio between the best Nash equilibrium and the optimum of the game.}