Chapter 6

Output SNR and array mismatch

This chapter discusses the signal-to-noise ratio (SNR) in the outputs of spatial filters. We first show that the adaptive spatial filters attain the maximum signal-to-interference-plus-noise ratio among all types of spatial filters. We next derive the SNR transfer factor, which is the ratio between the input and output SNRs, for several representative non-adaptive and adaptive spatial filters. We then show that a significant SNR degradation is caused in adaptive-spatial filter outputs by the array mismatch, which indicates a situation where the lead-field used for computing the spatial filter weight is different from the true lead field. We describe two kinds of techniques, diagonal loading and eigenspace-projection, that can reduce the SNR degradation caused by the array mismatch.

6.1 Output SINR

Let us consider the case where \( Q \) uncorrelated sources exist and the measured data contains external interference in addition to the sensor noise. Assuming that this interference is uncorrelated with the signal of interest, the measurement covariance matrix \( R \) is given by:

\[
R = \sum_{q=1}^{Q} \sigma_q^2 l(r_q)l^T(r_q) + R_{i+n}, \tag{6.1}
\]

where \( R_{i+n} \) indicates the interference-plus-noise covariance matrix, which is the covariance matrix only for the interference and the sensor noise. Let us next consider the case where, among the \( Q \) sources, we are attempting to reconstruct the first source, which is located at \( r_1 \). In such a case, the source at \( r_1 \) is the signal source or the source of interest and all other sources should be considered interference sources. Therefore, in such cases, the interference-plus-noise covariance
matrix should include the contributions from the sources located at \( r_2 \) to \( r_Q \). The contribution is equal to \( \sum_{q=2}^{Q} \sigma_q^2 l(r_q)l^T(r_q) \), and equation (6.1) is rewritten as

\[
R = \sigma_1^2 l(r_1)l^T(r_1) + R_{i+n}^{ex}(r_1),
\]

where

\[
R_{i+n}^{ex}(r_1) = \sum_{q=2}^{Q} \sigma_q^2 l(r_q)l^T(r_q) + R_{i+n}.
\]

Here, \( R_{i+n}^{ex} \) is called the extended interference-plus-noise covariance matrix. Note that this \( R_{i+n}^{ex} \) depends on the spatial-filter pointing location, which is \( r_1 \) in Eqs. (6.2) and (6.3), and we write \( R_{i+n}^{ex} \) as \( R_{i+n}^{ex}(r_1) \) to express this spatial-location dependency explicitly. When the pointing location \( r \) is equal to none of the source locations, \( R_{i+n}^{ex}(r) \) is equal to the covariance matrix \( R \).

When a source is located at \( r_1 \) and the spatial filter is pointing at this location, substituting Eq. (6.2) into (2.66) produces the power of the output of the spatial filter, which is expressed as

\[
\langle \hat{s}(r_1, t)^2 \rangle = w^T(r_1)Rw(r_1) = \sigma_1^2 \| w^T(r_1)l(r_1) \|^2 + w^T(r_1)R_{i+n}^{ex}(r_1)w(r_1).
\]

The first term on the right-hand side represents the power of the signal at \( r_1 \) and the second term represents the contribution from noise, interference, and other sources. The ratio between the first and the second terms on the right-hand side of Eq. (6.4) is denoted \( Z_0 \), i.e.,

\[
Z_0 = \frac{\sigma_1^2 \| w^T(r_1)l(r_1) \|^2}{w^T(r_1)R_{i+n}^{ex}(r_1)w(r_1)},
\]

This \( Z_0 \) is called the signal-to-interference-plus-noise ratio (SINR), which plays an important role in the performance analysis of adaptive spatial filters. In Section 6.2, we show that the minimum-variance spatial filters maximize this signal-to-interference-plus-noise ratio.

This \( Z_0 \) can be used for theoretical analysis. However, we cannot use \( Z_0 \) to evaluate the SNR of the spatial filter output in actual measurements, because \( \sigma_1^2 \) and \( R_{i+n}^{ex} \) are unknown. The numerator in Eq. (6.5) can be replaced with \( w^T(r)Rw(r) \), assuming that \( \sigma_1^2 \| w^T(r_1)l(r_1) \|^2 \gg w^T(r_1)R_{i+n}^{ex}(r_1)w(r_1) \) in Eq. (6.4). In the denominator of Eq. (6.5), the noise and interference covariance matrix, \( R_{i+n}^{ex} \), is replaced with \( \sigma_0^2 I \) when no information regarding \( R_{i+n}^{ex} \) is available. We then derive

\[
Z = \frac{w^T(r)Rw(r)}{\sigma_0^2 \| w \|^2},
\]

which is equal to Eq. (2.68). This \( Z \) can be computed from measurements, although we should estimate the variance of the input noise \( \sigma_0^2 \).