3 Some Basic Equations of Continuum Mechanics

Creep mechanics is a part of continuum mechanics, like elasticity, plasticity, viscoelasticity, and viscoplasticity.

*Continuum Mechanics* is concerned with the mechanical behavior of *solids* and *fluids* on the macroscopic scale. It ignores the discrete nature of matter, and treats material as uniformly distributed throughout regions of space. It is then possible to define quantities such as density, displacement, velocity, etc., as continuous (or at least piecewise continuous) functions of position. This procedure is found to be satisfactory provided that we deal with bodies whose dimensions are large in comparison with the characteristic lengths (e.g.: interatomic spacings in a crystal, or mean free paths in a gas) on the microscopic scale.

Continuum mechanics can also be applied to a granular material such as sand, concrete or soil, provided that the dimensions of the regions considered are large compared with those of an individual grain.

The equations of continuum mechanics are of two main kinds. First, there are equations which apply equally to all materials. They describe universal physical laws, such as conservation of mass and energy. Second, there are equations characterizing the individual material and its reaction to applied loads; such equations are called *constitutive equations* (Chapter 4), since they describe the macroscopic behavior resulting from internal constitution of the particular materials.

In this Chapter, however, only the *kinematics* and the *concept of stress* should briefly be discussed.

3.1 Analysis of Deformation and Strain

This section is concerned with the kinematics of a continuous medium. Kinematics is the study of *motion* without regard to the forces which produce it. To describe the motion of a body, i.e., to specify the position of each particle at each instant we select a particular configuration of the body, for instance
the configuration of the body in its unloaded or undeformed state, and call this the *reference configuration* at the *reference time* \( t = 0 \). The set of coordinates \( a_i \), referred to fixed cartesian axes, uniquely determines a particle of the body and may be regarded as a label by which the particle can be identified for all time (Fig. 3.1).

![Fig. 3.1 Motion of a particle; reference \((t = 0)\) and current \((t > 0)\) configurations](image)

The motion of the body may now be described by specifying the position \( x_i \) of the particle \( a_i \) at time \( t > 0 \) in the form

\[
x_i = x_i(a_p, t), \quad i, p = 1, 2, 3.
\]  

(3.1a)

In other words: The place \( x_i \) is occupied by the body-point \( a_p \) at time \( t \). We assume that this function is differentiable with respect to \( a_p \) and \( t \) as many times as required.

Sometimes we desire to consider only two configurations of the body, an initial and a final configuration. We refer to the mapping from the initial to the final configuration as a *deformation* of the body. The *motion* of the body may be regarded as a one-parameter sequence of deformations.

The mapping (3.1a) has the unique inverse

\[
a_i = a_i(x_p, t), \quad i, p = 1, 2, 3
\]  

(3.1b)