Equivalence Knowledge Mass and Approximate Reasoning in $\mathcal{R}$–Logic $\mathcal{C}_{\mathcal{R}}$ (I)

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Abstract. By casting off the direct restriction of topological structure, this paper presents another matching scheme between the input $A^*$ and the knowledge $A \rightarrow B$ based on the equivalence relation $\mathcal{R}$ on formulae set $\mathcal{F}(S)$ and the corresponding equivalence classification

$$\mathcal{F}(S)/\mathcal{R} = \{[A]_{\mathcal{R}} \mid A \in \mathcal{F}(S)\}$$

therefore, obtains another algorithm of approximate reasoning — the IV-type $\mathcal{R}$–algorithm.

Keywords: Knowledge Cumularspharolith, Formulae Cumularspsharolith, Approximate Knowledge Closure of Knowledge Base, Pseudo-distance, $\mathcal{R}$–automatic Reasoning System, $\mathcal{R}$–completeness.

1 Introduction

Many logistician have done a lot of research work for providing the logical foundation and logical normalization of the approximate reasoning. Guo-Jun Wang constructed a fuzzy propositional logic system $\mathcal{L}^*$ and gave the $\alpha - 3I$ algorithm of the approximate reasoning in [1]. In [2], Wang studied the logical metric space based on their integrate semantic theory. The approximate reasoning model in the framework of the classical propositional logic is also studied in [3]. Mingsheng Ying gave a logic model of approximate reasoning in [4] and studied approximate reasoning based on the fuzzy matching in [5]. We constructed the framework of the stratified fuzzy propositional logic in [6] and discovered an important logical property of Mamdanian algorithm for fuzzy reasoning in [7]. All the above work motivate the authors to study the approximate reasoning model in the framework of topological logic[8][9][10][11].
The key problem of the approximate reasoning that AI theories and techniques rely on is, how to approximate the matching between the input \( A^* \) and the knowledge \( A \rightarrow B \). As stated in [8-10], we have given three different kinds of schemes according to the different topological logic structures and their corresponding construction of the matching functions, thus we have given three different topological algorithms of the approximate reasoning, all of which agree in a certain sense with the common senses of human’s approximate reasoning. Recall these three algorithms, the key idea is based on such an understanding, that is, the topological structure can score rightly the similarity degree between one thing and another, therefore, it can be used to describe and regulate how the input \( A^* \) and the knowledge \( A \rightarrow B \) match. However, when we get to the bottom, we shall cognize at a higher level that, the essential of the topological algorithms and matching schemes we established is a group constituted by the following three formulae cumuluspharolithes

\[
(U_A, U_B, U_{A \rightarrow B}),
\]

which are specified for each implication \( A \rightarrow B \).

These formulae cumuluspharolithes

\[
(U_A, U_B, U_{A \rightarrow B})
\]

can be described with either topological method, as we have done before, or some other methods, as we will.

In this paper, we will cast off the direct restriction by topological structure, based on the equivalence relation \( R \) on formulae set \( F(S) \) and the corresponding equivalence classification

\[
F(S)/R = \{ [A]_R \mid A \in F(S) \}
\]

we propose another scheme of matching between the input \( A^* \) and the knowledge \( A \rightarrow B \)—— the IV-type matching scheme, therefore, obtain another algorithm of approximate reasoning —— the IV-type \( R- \)algorithm. According to this viewpoint, the equivalence replacement theorem in the classical proposition logic \( C \) becomes a special case of our IV-type \( R- \)algorithm for approximate reasoning.

2 Equivalence Relation \( R \), Pseudo-distance \( d_R \), Topology \( T_R \) on Formulae Set \( F(S) \) and Construction of \( R- \)Logic \( C_R \)

In classical propositional logic \( C = (\hat{C}, \tilde{C}) \), let \( R \) be an arbitrary equivalence relation on formulae set \( F(S) \), such that each implication \( A \rightarrow B \in \mathcal{H} \) satisfies

\[
[A]_R \rightarrow [B]_R \subseteq [A \rightarrow B]_R
\]

here,

\[
[A]_R \rightarrow [B]_R = \{ A^* \rightarrow B^* \mid A^* \in [A]_R, B^* \in [B]_R \}
\]