Computation with Imprecise Probabilities

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Extended Abstract

An imprecise probability distribution is an instance of second-order uncertainty, that is, uncertainty about uncertainty, or uncertainty2 for short. Another instance is an imprecise possibility distribution. Computation with imprecise probabilities is not an academic exercise – it is a bridge to reality. In the real world, imprecise probabilities are the norm rather than exception. In large measure, real-world probabilities are perceptions of likelihood. Perceptions are intrinsically imprecise, reflecting the bounded ability of human sensory organs, and ultimately the brain, to resolve detail and store information. Imprecision of perceptions is passed on to perceived probabilities. This is why real-world probabilities are, for the most part, imprecise.

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What is important to note is that in applications of probability theory in such fields as risk assessment, forecasting, planning, assessment of causality and fault diagnosis, it is a common practice to ignore imprecision of probabilities. The problem with this practice is that it leads to results whose validity is in doubt. This underscores the need for approaches in which imprecise probabilities are treated as imprecise probabilities rather than as precise probabilities.

Peter Walley’s seminal work “Statistical Reasoning with Imprecise Probabilities,” published in 1991, sparked a rapid growth of interest in imprecise probabilities. Today, we see a substantive literature, conferences, workshops and summer schools. An exposition of mainstream approaches to imprecise probabilities may be found in the 2002 special issue of the Journal of Statistical Planning and Inference (JSPI), edited by Jean-Marc Bernard. My paper “A perception-based theory of probabilistic reasoning with imprecise probabilities” (Zadeh 2002), is contained in this issue but is not a part of the mainstream. A mathematically rigorous treatment of elicitation of imprecise probabilities may be found in “A behavioural model for vague probability assessments,” by Gert de Cooman (2005).

The approach which is outlined in the following is rooted in my 1975 paper “The concept of a linguistic variable and its application to approximate reasoning” (Zadeh 1975), but in spirit it is close to my 2002 JSPI paper (Zadeh 2002). The approach is a radical departure from the mainstream. Its principal distinguishing features are: (a) imprecise probabilities are dealt with not in isolation, as in the mainstream approaches, but in an environment of imprecision of events, relations and constraints; (b) imprecise probabilities are assumed to be described in a natural language. This assumption is consistent with the fact that a natural language is basically a system for describing perceptions.

The capability to compute with information described in a natural language opens the door to consideration of problems which are not well-posed mathematically. Following are very simple examples of such problems.

1. $X$ is a real-valued random variable. What is known about $X$ is: (a) usually $X$ is much larger than approximately $a$; and (b) usually $X$ is much smaller than approximately $b$, with $a < b$. What is the expected value of $X$?

2. $X$ is a real-valued random variable. What is known is that $\text{Prob}(X \text{ is small})$ is low; $\text{Prob}(X \text{ is medium})$ is high; and $\text{Prob}(X \text{ is large})$ is low. What is the expected value of $X$?