Abstract. The proof theory of multi-agent epistemic logic extended with operators for distributed knowledge is studied. A proposition $A$ is distributed knowledge within a group $G$ if $A$ follows from the totality of what the individual members of $G$ know. There are known axiomatizations for epistemic logics with the distributed knowledge operator, but apparently no cut-free proof system for such logics has yet been presented. A Gentzen-style contraction-free sequent calculus system for propositional epistemic logic with operators for distributed knowledge is given, and a cut-elimination theorem for the system is proved. Examples of reasoning about distributed knowledge that use the calculus are given.

1 Introduction

Distributed knowledge is usually characterized by saying that $A$ is distributed knowledge within a group $G$ if $A$ follows from the totality of what the individual members of $G$ know. For instance, $A$ is distributed knowledge in group $G$ (denoted $D_G A$) consisting of three agents of which the first one knows $B$, the second one knows $B \supset C$, and the third one knows $B \& C \supset A$. Reasoning about the combined information possessed by different agents is an important task in multi-agent systems in which all information is not available in one central source but distributed among several agents.

In such situations, epistemic logic \cite{1} is typically used for representing and reasoning about knowledge. In the literature concerning multi-agent epistemic logics, e.g. \cite{2,3}, operators for distributed knowledge are often included. However, these treatments usually concentrate on the model theory of the logics, whereas the proof-theoretical part is limited to providing Hilbert-style axiomatizations. Since theorem-proving is difficult in Hilbert-style systems, we shall here study Gentzen-style sequent calculi as a step towards mechanization of proof search.

One proof-theoretical approach to reasoning about distributed knowledge is given in \cite{4}, but the approach is different because of the use of natural deduction instead of sequent calculus and context-based logic instead of epistemic logic. The development of a proof system for logic of distributed knowledge has been recently posed by S. Artemov as an open problem for the system of evidence-based knowledge (see \cite{5}). This paper presents a solution for ordinary multi-agent epistemic logic by the methods developed in \cite{6,7}.
Formal systems for drawing inferences in distributed knowledge may be useful in several application areas that attempt to combine knowledge of agents, such as cooperative problem solving, knowledge base merging, and judgement aggregation. In cooperative problem solving it is usually assumed that the agents are willing to provide any information they have and that all information is certain. In such situations, it is possible to combine the separate knowledge bases into one and then derive theorems from the large knowledge base. However, typically in knowledge base merging the data can contain errors, and is thus not strictly speaking knowledge. The combination of information from multiple sources may then lead to an inconsistent knowledge base, and special methods have to be used for dealing with contradictory information (see, e.g. [8,9,10]). These methods often involve discarding some information in order to maintain the integrity of the database.

In cases with heterogeneous information sources, the knowledge modalities should not be understood as knowledge proper but rather as beliefs. In open information systems, and in situations involving strategic considerations, such as in judgement aggregation or voting, agents can even provide false information on purpose, so it is not possible to infer their real beliefs from what they report, but the information they provide must be treated as claims, acceptances or just as messages with propositional content.

The introduction of the knowledge modalities and the modality for distributed knowledge into the logical language can be beneficial, because the management of the meta-information concerning the sources of knowledge and their various combinations becomes easier. When the source of information is stored in addition to the content, also contradictory information can be dealt with: If agent 1 claims that $A$ is the case and agent 2 claims that not-$A$ is the case, the receiving agent should decide which piece of information to accept and which one to reject. However, when such a situation arises there may not be enough information available for resolving the conflict. If our language is rich enough to allow also knowledge propositions and the agents are able to reason about distributed knowledge, incoming information need not be discarded nor is it necessary to immediately judge some agents unreliable. Instead, we can store the knowledge claims without violating the integrity constraints, and we can use the stored information to find out which agents we can trust, possibly later when we have gathered more information.

Thus, the addition of the knowledge operators to the language makes it possible for the agents to perform reasoning about the distributed information possessed by various agents and groups of agents and to detect inconsistencies between claims made by agents. Also, the possibility to iterate knowledge operators allows for more complex reasoning tasks than reasoning from an integrated knowledge base without iterated modalities. Reasoning of this type may be used in cooperative information systems to find out which agents have useful information with respect to the task at hand.

In Section 2, we introduce the logical system and show that it can be used to derive the axioms given in complete axiomatizations for the logic of distributed