Application of Fuzzy Theory to Binomial Option Pricing Model

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Abstract. This paper presents an extension of the binomial option pricing model, which has the capabilities to cope with uncertain assumptions. Such assumptions are represented and dealt with in the framework of fuzzy theory. As the stock price can not be known exactly in advance, the approach of taking stock price as fuzzy price is more realistic and be easily accepted. In this paper, we take stock price in each node as fuzzy variable instead of crisp, then build a fuzzy binomial tree model and get numerical result in one period case. The simulation for fuzzy multiperiod binomial pricing model is also provided.

Keywords: Options, Binomial option pricing, Fuzzy theory.

1 Introduction

As a kind of widely applied financial derivatives, options are play more and more important role in the financial market. Binomial options pricing model initiated by Cox, Ross and Rubinstein[5] which have simple structure is widely used in the financial market and become one of the basic options pricing methods. Researchers have so far made substantial effort and achieve significant results concerning the pricing of options, but these models [4, 9, 12] are not sufficient when the realistic uncertain environment is considered. Furthermore, jump factors are assumed as constant in those models. Empirical studies have shown that existing pricing model seem to have difficulty in properly handling the uncertainties inherent in any investment process. In reality, the future state of a system might not be known completely due to lack of information. Therefore, decision-making which based on individuals subjective perception or personality tendencies for judgment, evaluation and decisions are different in many areas. Investment problems are often are uncertain or vague in a number of ways, sometimes it is fuzziness.

Some researchers have got some results in binomial option pricing with fuzzy theory[16]. Lee, Tzeng and Wang [11] use fuzzy volatility and fuzzy riskless
interest rate to replace the corresponding crisp values. Muzzioli and Torricelli [14, 15] take both the risk-neutral probabilities and the stock price as weighted intervals. Appadoo et. al [2] presented a crisp riskless rate assisted by Capital Asset Pricing Model return in the fuzzy binomial option pricing model. Appadoo et. al [1, 3] used fuzzy sets to frame the binomial option pricing model. The pricing models of European option, which used the real interval limited Choquet integral for a nonnegative measurable function over a real fuzzy measure space, has been investigated by Kaino and Hirota. [10]. Han [6, 7, 8] also used the real interval limited Choquet integral for fuzzy option pricing.

In 2002, Liu and Liu [13] give the concept of credibility measure. Credibility theory is the branch of mathematics that studies the behavior of fuzzy events, which is based on fuzzy theory. In this paper, fuzzy theory is applied to traditional binomial model, jump factors are assumed as fuzzy variable, then the fuzzy binomial tree model can be built to price European options and American options. With the approach of eliminating the fuzziness in the result, the method of getting expected value is adopted to help investor make descion. In one period model, numerical result of expected value can be got and the membership function of option value is approximately given. Fuzzy simulation is available for multiperiod case. to standard binomial model in pricing European call options and get clear result by taking expected value (this model is also suits for American options).

The remainder of this paper is organized as follows. fuzzy theory is briefly introduced in section 2. In section 3, we apply fuzzy theory to standard binomial tree model in order to get option value by means of expectation. We can have numerical result in one-period model, and multiperiod case can calculated by simulation. The result calculated by the model also has economical meaning, which is proved by an example. The last section is a briefly conclusion.

2 Fuzzy Variables

Let \( \Theta \) be a nonempty set, and \( \mathcal{P}(\Theta) \) the power set of \( \Theta \). Each element in \( \mathcal{P}(\Theta) \) is called an event. \( \text{Cr} \) a credibility measure defined on \( \mathcal{P}(\Theta) \).

**Definition 1.** [13] A Fuzzy variable is defined as a function from a credibility space \( (\Theta, \mathcal{P}(\Theta), \text{Cr}) \) to the set of real number.

**Definition 2.** [13] Let \( \xi \) be a fuzzy variable with membership function \( \mu \), then for any set \( \mathcal{B} \) of real numbers,

\[
\text{Cr}\{\xi \in \mathcal{B}\} = \frac{1}{2} \left( 1 + \sup_{x \in \mathcal{B}} \mu(x) - \sup_{x \in \mathcal{B}^c} \mu(x) \right).
\]

**Definition 3.** [13] The credibility distribution \( \Phi : \mathbb{R} \to [0, 1] \) of a fuzzy variable \( \xi \) is defined by

\[
\Phi(x) = \text{Cr}\{\theta \in \Theta | \xi(\theta) \leq x\}.
\]