Chapter 3
Foundations of Metaheuristics

Abstract. Many real-world problems with practical importance are large and complex, differing from standard problems. Often they belong to the class $NP$ making them computationally intractable using exact optimization algorithms. Metaheuristics are general-purpose improvement heuristics that are used to solve $NP$-hard problems. Some problem-specific adaptations of the metaheuristic are necessary to achieve an efficient search process. This problem customization focuses on four basic design elements that every metaheuristic incorporates and that are presented in detail in this chapter: the combination of representation and search operators, the fitness function, the initialization, and the search strategy. With respect to the search strategy, in general the different variants of metaheuristics can be classified into two groups: techniques based on local search and techniques using recombination-based search operators. For each search concept, one metaheuristic was specified as a representative example: threshold accepting as a local search routine and genetic algorithms as a recombination-based search. They are the underlying techniques of the simultaneous airline scheduling process presented in this work.

3.1 Introduction

To reach a certain pre-defined goal, a decision maker has to choose between different decision alternatives. Planning describes this process of generating and comparing different courses of action and then choosing one of them prior to action (Rothlauf, 2006a). As each alternative might have a different impact on the decision maker’s objective, it is necessary to evaluate each given alternative with regard to reaching the goal and to assign a quality value. Furthermore, restrictions and limitations have to be taken into account that limit the freedom of action. One possible decision alternative is denoted as the solution of the problem, the complete set of possible solutions establishes the search space of the problem. In optimization problems, the objective is to find the best solution: the decision alternative with the highest contribution to the overall goal.
Planning processes and the solution of optimization problems are the core discipline of operations research (OR) (Taha, 2002; Hillier & Lieberman, 2002; Domschke & Drexl, 2005). Its focus is on model construction and model solution:

- **Model Construction:** The objective of this step is to simplify the given real-world problem to make it computationally tractable. The properties of the real-world problem, its influencing factors, and relationships are represented as a mathematical model consisting of a set of symbols and expressions. Decision variables represent the properties of the different decision alternatives. The quality of an alternative is calculated using an objective function. Any restrictions on the problem or its variables are expressed by a set of constraints.

- **Model Solution:** Given an optimization model, algorithms are applied to solve the model. Because many different solution techniques were developed in the past, this step is straightforward if an appropriate model was constructed, requiring only the choice of one suitable solution method. In fact, OR focuses on model construction which itself is demanding and requires the researcher’s experience. Sometimes, model solution is considered only as a (simple) subsequent step (Ackhoff, 1973).

Many of the standard optimization algorithms developed by OR are exact techniques that guarantee to find the optimal solution of the given problem. If their effort grows polynomially with the problem size (P-problems), exact optimization algorithms can be applied. However, many problems belong to the class $NP$. Problems of this class are less structured than P-problems (for example, the objective function is not differentiable, the objective function and constraints and variables are not linear and continuous etc. (Rudolph & Schwefel, 1994)). Then, enumeration techniques are the only exact algorithms that can be used to solve these problems. As the effort for problem solution grows exponentially with problem size, these techniques can only be applied if the problem size is low enough.

### 3.2 Metaheuristic Optimization

Many real-world problems with practical importance are large and complex, differing from standard problems. Often they belong to the class $NP$ making them computationally intractable using exact optimization algorithms. To solve these kinds of problems, heuristic optimization techniques could be used. Heuristics often exploit properties of the problem and use knowledge about high-quality solutions or rules of thumb when searching for good solutions (Pearl, 1984). They often have lower computation times than exact techniques allowing their application to complex problems that are realistic and near to real-world problems. However, they do not guarantee to find the optimal solution. In general, there are two kinds of heuristics: construction heuristics develop a solution to a problem by iteratively adding elements to partial solutions until a complete solution is achieved, whereas improvement heuristics start with a complete solution and iteratively improve the solution by slight modifications.