Chapter 3

Acceptability Index and Interval Linear Programming

3.1. Introduction

This chapter defines an interval linear programming problem as an extension of the classical linear programming problem to an inexact environment. Let’s refer here a very good example (Tong (1994)) of using interval numbers in an optimization problem:

There are 1000 chickens raised in a chicken farm and they are raised with two kinds of forages – soya and millet. It is known that each chicken eats 1 – 1.3 kg of forage every day and that for good weight gain it needs at least 0.21 – 0.23 kg of protein and 0.004 – 0.006 kg of calcium everyday. Per kg of soya contains 48 – 52% protein and 0.5 – 0.8% calcium and its price is 0.38 – 0.42 Yuan. Per kg of millet contains 8.5 – 11.5% protein and 0.3% calcium and its price is 0.20 Yuan. How should the forage be mixed in order to minimize expense on forage?

Most of the parameters used in this problem are inexact and perhaps appropriately given in terms of simple intervals. In reality inexactness of this kind can be

The optimization problem can be structured as follows:

Minimize $Z = [0.38, 0.42] x_1 + 0.20 x_2$,

subject to

$x_1 + x_2 = [1, 1.3] \times 1000,$

$[0.48, 0.52] x_1 + [0.085, 0.115] x_2 \geq [0.21, 0.23] \times 1000,$

$[0.005, 0.008] x_1 + 0.003 x_2 \geq [0.004, 0.006] \times 1000,$

$x_1 \geq 0, \quad x_2 \geq 0.$

However, for solution, the techniques of classical linear programming cannot be applied if and unless the above interval-valued structure of the problem is reduced into a standard linear programming structure (Luhandjula (1986), Rommelfanger (1989), Chakraborty (1995)) and for that we have to clear up the following main issues:

– First, regarding interpretation and realization of the inequality relations involving interval coefficients.

– Second, regarding interpretation and realization of the objective ‘Minimize’ with respect to an inexact environment.

In this chapter, we concentrate mainly on a satisfactory solution approach which needs DM’s interpretation regarding the inequality relations and the objective of the problem defined in an inexact environment.

This chapter is organized as follows. In Section 3.2, we introduce acceptability index, or $A$-index in short, for comparing any two interval-numbers. Various properties of this index and an illustrative example have been discussed here. Section 3.3 divided in three subsections gives an elaborate study on the interval inequality relation in order to interpret and realize the relation as an interval-valued constraint of an optimization problem defined in an inexact environment. On the basis of $A$-index, inequality and equality constraints involving interval coefficients are reduced to their satisfactory crisp equivalent forms. Section 3.4 describes the solution principle of an interval linear programming problem, and discusses efficiency of our methodology in comparison to a previously cited numerical problem (Tong (1994)). Section 3.5 includes the concluding remarks and the future scope.