8

Dielectric Properties of Solids

8.1 Review of Some Ideas of Electricity and Magnetism

When an external electromagnetic disturbance is introduced into a solid, it will produce induced charge density and induced current density. These induced densities produce induced electric and magnetic fields. We begin with a brief review of some elementary electricity and magnetism. In this chapter, we will neglect the magnetization produced by induced current density and concentrate on the electric polarization field produced by the induced charge density.

The potential \( \phi(\mathbf{r}) \) set up by a collection of charges \( q_i \) at positions \( \mathbf{r}_i \) is given by

\[
\phi(\mathbf{r}) = \sum_i \frac{q_i}{|\mathbf{r} - \mathbf{r}_i|}. \tag{8.1}
\]

The electric field \( \mathbf{E}(\mathbf{r}) \) is given by \( \mathbf{E}(\mathbf{r}) = -\nabla \phi(\mathbf{r}) \).

Now, consider a dipole at position \( \mathbf{r}' \) (see Fig. 8.1).

\[
\phi(\mathbf{r}) = \frac{q}{|\mathbf{r} - \mathbf{r}' - \frac{\mathbf{d}}{2}|} - \frac{q}{|\mathbf{r} - \mathbf{r}' + \frac{\mathbf{d}}{2}|}. \tag{8.2}
\]

By a dipole we mean \( \mathbf{p} = q\mathbf{d} \) is a constant, called the dipole moment, but \( |\mathbf{d}| = d \) itself is vanishingly small. If we expand for \( |\mathbf{r} - \mathbf{r}'| \gg |\mathbf{d}| \), we find

\[
\phi(\mathbf{r}) = \frac{q\mathbf{d} \cdot (\mathbf{r} - \mathbf{r}')}{(\mathbf{r} - \mathbf{r}')^3} = \frac{\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}. \tag{8.3}
\]

The potential produced by a collection of dipoles \( \mathbf{p}_i \) located at \( \mathbf{r}_i \) is simply

\[
\phi(\mathbf{r}) = \sum_i \frac{\mathbf{p}_i \cdot (\mathbf{r} - \mathbf{r}_i)}{|\mathbf{r} - \mathbf{r}_i|^3}. \tag{8.4}
\]

Again the electric field \( \mathbf{E}(\mathbf{r}) = -\nabla \phi(\mathbf{r}) \), so

\[
\mathbf{E}(\mathbf{r}) = \sum_i \frac{3(\mathbf{r} - \mathbf{r}_i) [\mathbf{p}_i \cdot (\mathbf{r} - \mathbf{r}_i)] - (\mathbf{r} - \mathbf{r}_i)^2 \mathbf{p}_i}{|\mathbf{r} - \mathbf{r}_i|^5}. \tag{8.5}
\]
8.2 Dipole Moment Per Unit Volume

Let us introduce the electric polarization \( \mathbf{P}(\mathbf{r}) \), which is the dipole moments per unit volume. Consider a volume \( V \) bounded by a surface \( S \) filled with a polarization \( \mathbf{P}(\mathbf{r}') \) that depends on the position \( \mathbf{r}' \). Then

\[
\phi(\mathbf{r}) = \int d^3 r' \frac{\mathbf{P}(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}. \tag{8.6}
\]

If we look at the divergence of \( \frac{\mathbf{P}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \) with respect to \( \mathbf{r}' \), we note that

\[
\nabla' \cdot \left[ \frac{\mathbf{P}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \right] = \frac{1}{|\mathbf{r} - \mathbf{r}'|} \nabla' \cdot \mathbf{P}(\mathbf{r}') + \frac{\mathbf{P}(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}. \tag{8.7}
\]

We can solve for \( \frac{\mathbf{P}(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \) and substitute into our expression for \( \phi(\mathbf{r}) \). The integral of the divergence term can be expressed as a surface integral using divergence theorem. This gives

\[
\phi(\mathbf{r}) = \oint_S dS' \frac{\mathbf{P}(\mathbf{r}') \cdot \hat{n}'}{|\mathbf{r} - \mathbf{r}'|} + \int_V d^3 r' \frac{[-\nabla' \cdot \mathbf{P}(\mathbf{r}')]}{|\mathbf{r} - \mathbf{r}'|}. \tag{8.8}
\]

The potential \( \phi(\mathbf{r}) \) can be associated with a potential produced by a volume distribution of charge density

\[
\rho_P(\mathbf{r}) = -\nabla \cdot \mathbf{P}(\mathbf{r}) \tag{8.9}
\]

and the potential produced by a surface charge density

\[
\sigma_P(\mathbf{r}) = \mathbf{P}(\mathbf{r}) \cdot \hat{n}. \tag{8.10}
\]

Here, of course, \( \hat{n} \) is a unit vector outward normal to the surface \( S \) bounding the volume \( V \).