Chapter 4
Interaction between Central Bank and Government B

1. The Model

This chapter deals with case B. The targets of the European central bank are zero inflation and zero unemployment. The targets of the European government are zero unemployment and a zero structural deficit. The model of unemployment, inflation, and the structural deficit can be characterized by a system of three equations:

\begin{align*}
    u &= A - M - G \\
    \pi &= B + M + G \\
    s &= G - T
\end{align*}

The targets of the European central bank are zero inflation and zero unemployment in Europe. The instrument of the European central bank is European money supply. There are two targets but only one instrument, so what is needed is a loss function. We assume that the European central bank has a quadratic loss function:

\begin{equation}
    L_1 = \pi^2 + u^2
\end{equation}

$L_1$ is the loss to the European central bank caused by inflation and unemployment. We assume equal weights in the loss function. The specific target of the European central bank is to minimize the loss, given the inflation function and the unemployment function. Taking account of equations (1) and (2), the loss function of the European central bank can be written as follows:

\begin{equation}
    L_1 = (B + M + G)^2 + (A - M - G)^2
\end{equation}
Then the first-order condition for a minimum loss gives the reaction function of
the European central bank:

\[ 2M = A - B - 2G \]  

(6)

Suppose the European government raises European government purchases. Then,
as a response, the European central bank lowers European money supply.

The targets of the European government are zero unemployment and a zero
structural deficit in Europe. The instrument of the European government is
European government purchases. There are two targets but only one instrument,
so what is needed is a loss function. We assume that the European government
has a quadratic loss function:

\[ L_2 = u^2 + s^2 \]  

(7)

\( L_2 \) is the loss to the European government caused by unemployment and the
structural deficit. We assume equal weights in the loss function. The specific
target of the European government is to minimize the loss, given the
unemployment function and the structural deficit function. Taking account of
equations (1) and (3), the loss function of the European government can be
written as follows:

\[ L_2 = (A - M - G)^2 + (G - T)^2 \]  

(8)

Then the first-order condition for a minimum loss gives the reaction function of
the European government:

\[ 2G = A + T - M \]  

(9)

Suppose the European central bank lowers European money supply. Then, as a
response, the European government raises European government purchases.

The Nash equilibrium is determined by the reaction functions of the
European central bank and the European government. The solution to this
problem is as follows: