Chapter 4  
Interaction between European Central Bank, German Government, and French Government B

1. The Model

This chapter deals with case B. The targets of the European central bank are zero inflation and zero unemployment in each of the member countries. The targets of the German government are zero unemployment and a zero structural deficit in Germany. And the targets of the French government are zero unemployment and a zero structural deficit in France.

The model of unemployment, inflation, and the structural deficit can be characterized by a system of six equations:

\[ u_1 = A_1 - M - G_1 \]  
(1)  
\[ u_2 = A_2 - M - G_2 \]  
(2)  
\[ \pi_1 = B_1 + M + G_1 \]  
(3)  
\[ \pi_2 = B_2 + M + G_2 \]  
(4)  
\[ s_1 = G_1 - T_1 \]  
(5)  
\[ s_2 = G_2 - T_2 \]  
(6)  

The targets of the European central bank are zero inflation and zero unemployment in each of the member countries. The instrument of the European central bank is European money supply. There are four targets but only one instrument, so what is needed is a loss function. We assume that the European central bank has a quadratic loss function:

\[ L = \pi_1^2 + \pi_2^2 + u_1^2 + u_2^2 \]  
(7)
L is the loss to the European central bank caused by inflation and unemployment in each of the member countries. We assume equal weights in the loss function. The specific target of the European central bank is to minimize the loss, given the inflation functions and the unemployment functions. Taking account of equations (1) to (4), the loss function of the European central bank can be written as follows:

\[
L = (B_1 + M + G_1)^2 + (B_2 + M + G_2)^2 \\
+ (A_1 - M - G_1)^2 + (A_2 - M - G_2)^2
\]

(8)

Then the first-order condition for a minimum loss gives the reaction function of the European central bank:

\[
4M = A_1 + A_2 - B_1 - B_2 - 2G_1 - 2G_2
\]

(9)

Suppose the German government raises German government purchases. Then, as a response, the European central bank lowers European money supply.

The targets of the German government are zero unemployment and a zero structural deficit in Germany. The instrument of the German government is German government purchases. There are two targets but only one instrument, so what is needed is a loss function. We assume that the German government has a quadratic loss function:

\[
L_1 = u_1^2 + s_1^2
\]

(10)

\[L_1\] is the loss to the German government caused by unemployment and the structural deficit in Germany. We assume equal weights in the loss function. The specific target of the German government is to minimize the loss, given the unemployment function and the structural deficit function. Taking account of equations (1) and (5), the loss function of the German government can be written as follows:

\[
L_1 = (A_1 - M - G_1)^2 + (G_1 - T_1)^2
\]

(11)