Chapter 6
Cooperation between Central Bank and Government

1. The Model

An increase in European money supply lowers unemployment in Europe. On the other hand, it raises inflation there. Correspondingly, an increase in European government purchases lowers unemployment in Europe. On the other hand, it raises inflation there. The policy makers are the European central bank and the European government. The targets of policy cooperation are zero inflation and zero unemployment in Europe.

The model of unemployment and inflation can be characterized by a system of two equations:

\[ u = A - \alpha M - \beta G \]  
\[ \pi = B + \alpha \epsilon M + \beta \epsilon G \]

Of course this is a reduced form. Here \( u \) denotes the rate of unemployment in Europe, \( \pi \) is the rate of inflation in Europe, \( M \) is European money supply, \( G \) is European government purchases, \( \alpha \) is the monetary policy multiplier with respect to unemployment, \( \alpha \epsilon \) is the monetary policy multiplier with respect to inflation, \( \beta \) is the fiscal policy multiplier with respect to unemployment, \( \beta \epsilon \) is the fiscal policy multiplier with respect to inflation, \( A \) is some other factors bearing on the rate of unemployment in Europe, and \( B \) is some other factors bearing on the rate of inflation in Europe. The endogenous variables are the rate of unemployment and the rate of inflation in Europe.

According to equation (1), the rate of unemployment in Europe is a positive function of \( A \), a negative function of European money supply, and a negative function of European government purchases. According to equation (2), the rate of inflation in Europe is a positive function of \( B \), a positive function of European government purchases, and a

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money supply, and a positive function of European government purchases. A unit increase in A raises the rate of unemployment by 1 percentage point. A unit increase in B raises the rate of inflation by 1 percentage point. A unit increase in money supply lowers the rate of unemployment by $\alpha$ percentage points. On the other hand, it raises the rate of inflation by $\alpha \varepsilon$ percentage points. A unit increase in government purchases lowers the rate of unemployment by $\beta$ percentage points. On the other hand, it raises the rate of inflation by $\beta \varepsilon$ percentage points.

The policy makers are the European central bank and the European government. The targets of policy cooperation are zero inflation and zero unemployment in Europe. The instruments of policy cooperation are European money supply and European government purchases. Thus there are two targets and two instruments. We assume that the policy makers agree on a common loss function:

$$L = \pi^2 + u^2$$

$L$ is the loss caused by inflation and unemployment. For ease of exposition we assume equal weights in the loss function. The specific target of policy cooperation is to minimize the loss, given the inflation function and the unemployment function. Taking account of equations (1) and (2), the loss function under policy cooperation can be written as follows:

$$L = (B + \alpha \varepsilon M + \beta \varepsilon G)^2 + (A - \alpha M - \beta G)^2$$

Then the first-order conditions for a minimum loss are:

$$(1 + \varepsilon^2)\alpha M = A - \varepsilon B - (1 + \varepsilon^2)\beta G$$

$$(1 + \varepsilon^2)\beta G = A - \varepsilon B - (1 + \varepsilon^2)\alpha M$$

Equation (5) shows the first-order condition with respect to European money supply. And equation (6) shows the first-order condition with respect to European government purchases. Obviously, equations (5) and (6) are identical. There are two endogenous variables, European money supply and European government purchases. On the other hand, there is only one independent equation. Thus there is an infinite number of solutions.