Chapter 1
Waves and Instabilities in Space Plasmas

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1.1 Introduction

Electromagnetic plasma waves are a ubiquitous feature of space plasmas. They are important agents in propagating energy across different space regions, in providing plasma transport in the absence of collisions in the form of anomalous resistivity, viscosity and isotropization, in accelerating particles to high energies, and in transmitting diagnostic information for the local plasma properties from regions not accessible to in situ measurements. Waves are generated by thermal and non-thermal particle distributions of the plasma populations, by spontaneous or stimulated emission or by instabilities driven by free energy sources in the plasma.

This tutorial lecture does not attempt to provide a comprehensive coverage of the topic that has been the subject of numerous books and articles [2, 22, 6, 21, 10, 7, 9, 3]. It is an eclectic review of wave processes of importance to space plasmas; it reflects my personal style as a practitioner of plasma physics in various space and laboratory settings. The style emphasizes simplicity and physics intuition over strict mathematical rigor (that I liked to leave to my graduate students!). An important ingredient of the tutorial is a parallel exposition of the basic plasma characteristics of the modes, including polarization, phase and group velocities, refractive index surfaces, interaction with particles, mode conversion and transport properties, weak and strong turbulence theories, coupled to observations from space and the laboratory and computer simulations. Emphasis is placed on concepts rather than detailed analysis. Thus most of the review deals with unmagnetized or equivalently weakly magnetized plasmas. Extension to magnetized plasmas is straightforward but laborious.
1.2 Plasma Description – Response Function – Generalized Ohm’s Law

The basic set of equations that provides a complete description of waves in a plasma is

\[
\nabla \times \mathbf{B}(r, t) = \mu_0 [J_0(r, t) + J_p(r, t)] + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}(r, t)}{\partial t},
\]

(1.1)

\[
\nabla \times \mathbf{E}(r, t) = -\frac{\partial \mathbf{B}(r, t)}{\partial t},
\]

(1.2)

\[
\nabla \cdot \mathbf{B}(r, t) = 0,
\]

(1.3)

\[
\nabla \cdot \mathbf{E}(r, t) = \frac{1}{\varepsilon_0} [\rho_0(r, t) + \rho_p(r, t)],
\]

(1.4)

where \( J_0(r, t) \) and \( \rho_0(r, t) \) are current and charge densities due to external sources (e.g., coils, and internal currents of the earth or planets) and \( J_p(r, t) \) and \( \rho_p(r, t) \) are current and charge densities induced in the plasma.

It is important to note the physical meaning of the electric and magnetic field \( \mathbf{E}(r, t) \) and \( \mathbf{B}(r, t) \). In both vacuum and inside plasma the fields are defined by the force they exert on a test charge \( e \) moving with velocity \( \mathbf{V} \), i.e.,

\[
\mathbf{F}(r, t) = e[\mathbf{E}(r, t) + \mathbf{V} \times \mathbf{B}(r, t)].
\]

(1.5)

As in all traditional electrodynamics only the first two of Maxwell’s equations are required, while the last two are essentially initial conditions that provide self-consistency at \( t = 0 \). Furthermore, only the current densities are required since the charge densities can be found from the continuity of charge equation.

In order to solve plasma wave problems we need a model that expresses \( J_p(r, t) \) as a function of the fields \( \mathbf{E}(r, t) \) and \( \mathbf{B}(r, t) \). The relationship \( J_p(r, t) = f(\mathbf{E}, \mathbf{B}) \) is called the internal response function and does not have to be a linear function of \( \mathbf{E} \) and \( \mathbf{B} \), although most often is taken as linear. A major part of plasma physics is devoted to the development and justification of plasma models that allow us to compute the plasma currents that are created by the external excitation of an electric or magnetic field. For any given plasma model a self-consistent description can be developed that, at least in principle, solves the problem of waves in plasmas.

A common approach to the computation of the plasma response function is to introduce a conductivity tensor \( \tilde{\sigma} \) that connects the plasma current density to the electric field that drives it. It is a generalized Ohm’s law. The most general form of the conductivity tensor should be \textbf{non-linear} in \( E \) and non-local in space–time (i.e., it should express the dependence of the plasma current on the space–time history of the electric field). However, here as in most plasma wave analysis, we restrict ourselves to situations that the conductivity tensor is independent of the electric field amplitude. This is called the \textbf{small signal or linear theory}. In this case, the generalized Ohm’s law is written in the following form: