Control of Flexible Rotors

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The goal of this chapter is to discuss the problems that rotor flexibility and hardware limitations introduce in the design of AMB controllers and to present some solution strategies for these problems. Rotor flexibility means that the rotor can have relatively high gain at higher frequencies and this introduces complications in designing controllers with physically realizable bandwidths. Further, non-collocation of actuators and sensors along with finite bandwidth of actuation, sensing, and control mechanisms can mean that a passivity type of approach to controller design is not feasible.

These issues and others will be explored here through a series of examples. Control solutions are presented for a flexible rotor ranging from the simplest PID approach through to a fairly sophisticated $\mu$—synthesis solution. The performances of these controllers are compared in terms of complexity, forced response performance, and sensitivity to model parameters.

The literature relating to control of AMBs and, especially, those supporting flexible rotors is vast: certainly the largest segment of AMB literature is devoted to control. The bibliography for this chapter attempts to provide a survey of this literature but is by no means complete: a comprehensive survey would contain hundreds of references. Over 80 references are provided: a mix of background material on the general control problem and papers directed specifically at AMB control.

12.1 Flexibility Effects

There are two reasons why flexible systems present more of a challenge to the control system designer than does a rigid rotor. The first is the simple matter that a flexible rotor has a much wider mechanical bandwidth than does a rigid rotor. This means that the mechanical response to high frequency forcing is much larger for a flexible rotor than for a rigid rotor and, as a result, the dynamic behavior of the feedback controller at high frequencies is much more important for flexible rotors than for rigid rotors. The second reason is that,
when the sensors and actuators are not collocated axially along the rotor, there will always be flexible modes with a node between a sensor-actuator pair. If these modes have frequencies within the bandwidth of the controller, then they pose special dynamics problems for the system. Both of these issues must be attended to either explicitly or implicitly in the design of an AMB controller for a flexible rotor.

To illustrate these problems in a simple way, consider the control problem posed by a flexible beam that has a pinned support at one end so there is only one axis of control: this is illustrated in Fig. 12.1. This arrangement eliminates the complicating effects of interaction between control axes normally encountered in a fully levitated rotor while still exhibiting the bandwidth and non-collocation problems of a flexible rotor.

![Fig. 12.1. A pinned beam controlled at the free end by an active magnetic bearing.](image)

Denote the transfer function from actuator location input to sensor location output as $G_r(s)$. For a rigid rotor, this transfer function is

$$G_r(s) = \frac{y_s(s)}{f_a(s)} = \frac{\lambda}{s^2}$$  \quad (12.1)

and has the frequency response plot indicated in Fig. 12.2. The value of $\lambda$ depends both on the mass of the rotor and on the locations of the sensor and actuator. For a slender uniform cylinder with mass per unit length $\rho A$, total length $L$, sensor located at $x_s$ and actuator located at $x_a$,

$$\lambda \approx \frac{3x_ax_s}{\rho AL^3} , \; A \ll L^2$$

Moving the sensor or the actuator along the beam only changes the gain of the plant transfer function, not its dynamic character - in this case, its poles or eigenvalues.

A plant with such a simple transfer function can readily be stabilized using a phase lead controller:

$$C(s) = k \frac{s + z}{s + \beta z} : \; z > 0, \; \beta > 1$$  \quad (12.2)