Problem Solving for the 21st Century

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Mathematical problem solving has been the subject of substantial and often controversial research for several decades. We use the term, problem solving, here in a broad sense to cover a range of activities that challenge and extend one’s thinking. In this chapter, we initially present a sketch of past decades of research on mathematical problem solving and its impact on the mathematics curriculum. We then consider some of the factors that have limited previous research on problem solving. In the remainder of the chapter we address some ways in which we might advance the fields of problem-solving research and curriculum development.

A Brief Reflection on Problem-Solving Research

In this section, we do not attempt to provide a comprehensive coverage of problem-solving research over past decades. There are several other sources that provide such coverage, including Lester and Kehle’s (2003) work on the development of thinking about research on complex mathematical activity, Lesh and Zawojewski’s (2007) research on problem solving and modeling, and English and Halford’s (1995) work on problem solving, problem posing, and mathematical thinking.

Concerns about students’ mathematical problem solving can be traced back as far as the period of meaningful learning (1930s and 1940s), where William Brownell (1945), for example, emphasized the importance of students appreciating and understanding the structure of mathematics. In a similar vein, Van En gen (1949) stressed the need to develop students’ ability to detect patterns in similar and seemingly diverse situations. However, it was Polya’s (1945) seminal work on how to solve problems that provided the impetus for a lot of problem-solving research that took place in the following decades. Included in this research have been studies on computer-simulated problem solving (e.g., Simon 1978), expert problem solving (e.g., Anderson et al. 1985), problem solving
strategies/heuristics and metacognitive processes (e.g., Charles and Silver 1988; Lester et al. 1989), and problem posing (Brown and Walter 2005; English 2003). More recently there has been an increased focus on mathematical modeling in the elementary and middle grades, as well as interdisciplinary problem solving (English 2009a). The role of complexity and complex systems in the mathematics curriculum is just starting to be explored (e.g., Campbell 2006; Davis and Simmt 2003; English 2007; Lesh 2006), as is the role of educational neuroscience in helping us improve students’ mathematics learning (Campbell 2006).

A sizeable proportion of past research has focused primarily on word problems of the type emphasized in school textbooks or tests. These include “routine” word problems requiring application of a standard computational procedure, as well as “non-routine” problems involving getting from a given to a goal when the path is not evident. It is the latter problems with which students especially struggled. Polya’s book, How to Solve It (1945), was thus a welcomed publication because it introduced the notion of heuristics and strategies—such as work out a plan, identify the givens and goals, draw a picture, work backwards, and look for a similar problem—tools of an “expert” problem solver. Mathematics educators seized upon the book, viewing it as a valuable resource for improving students’ abilities to solve unfamiliar problems, that is, to address the usual question of “What should I do when I’m stuck?”

Despite the ground-breaking contribution of Polya’s book, it seems that the teaching of heuristics and strategies has not made significant inroads into improving students’ problem solving (Lesh and Zawojewski 2007; Schoenfeld 1992; Silver 1985). Even back in 1979, Begle noted in his seminal book, Critical Variables in Mathematics Education:

A substantial amount of effort has gone into attempts to find out what strategies students use in attempting to solve mathematical problems . . . no clear-cut directions for mathematics education are provided. . . In fact, there are enough indications that problem-solving strategies are both problem- and student-specific often enough to suggest that hopes of finding one (or few) strategies which should be taught to all (or most) students are far too simplistic. (p. 145)

Six years later, Silver’s (1985) report was no more encouraging. His assessment of the literature showed that, even in studies where some successful learning had been reported, transfer of learning was unimpressive. Furthermore, improvement in problem solving usually occurred only when expert teachers taught lengthy and complex courses in which the size and complexity of the interventions made it unclear exactly why performance had improved. Silver suggested that these improvements could have resulted simply from the students learning relevant mathematical concepts, rather than from learning problem-solving strategies, heuristics, or processes.

Seven years on, Schoenfeld’s (1992) review of problem-solving research also concluded that attempts to teach students to apply Polya-style heuristics and strategies generally had not proven to be successful. Schoenfeld suggested that one reason for this lack of success could be because many of Polya’s heuristics appear to be “descriptive rather than prescriptive” (p. 353). That is, most are really just names for