Chapter 9
DC Motor Starting Phase

This chapter is aimed at showing that, even for DC motor control, a quite standard application of control, described by a single input linear system, the so-called flatness-based approach may dramatically improve its performance in a transient phase.

More precisely, we consider a DC motor on which we test two control laws starting at rest to reach a given permanent angular speed in a given duration.

The first control law is a classical PID controller with a step speed reference, and the second one is the same PID controller with a flatness-based reference trajectory in place of the step reference, in order to evaluate the impact of the feedforward design.

Fig. 9.1 A standard DC motor
The motor has inductance $L$, resistance $R$, assumed constant, and torque constant $K$. Its inertia is denoted by $J$ and its coefficient of viscous friction $K_v$. An unknown resistive torque $C_r$ is applied to the motor. The current through the motor is denoted by $I$, the angular speed by $\omega$ and the input voltage by $U$, the control variable.

At the initial time $t_i$ we have $\omega(t_i) = \dot{\omega}(t_i) = 0$, and, at the final time $t_f$ (the duration is noted $T = t_f - t_i$), we want the motor to reach the angular speed $\omega(t_f) = \omega_f$ with $\dot{\omega}(t_f) = 0$.

The model is given by

$$L \frac{dI}{dt} = U - RI - K\omega \quad (9.1)$$
$$J \frac{d\omega}{dt} = KI - K_v\omega - C_r.$$

This system is indeed flat with $\omega$ as flat output: setting $y = \omega$, all the system variables can be expressed as functions of $y$ and derivatives up to second order:

$$\omega = y$$
$$I = \frac{1}{K} (J\dot{y} + K_v y + C_r)$$
$$U = L \frac{dI}{dt} + RI + Ky = \frac{K^2}{K} + R K_v y + \frac{R J + L K_v}{K} \dot{y} + \frac{J L}{K} \ddot{y} + \frac{R}{K} C_r + \frac{L}{K} \dot{C}_r. \quad (9.2)$$

### 9.1 Tracking of a Step Speed Reference

A classical approach consists in choosing as reference trajectory of $\omega$ the speed step

$$\omega^*(t) = \begin{cases} 
0 & \text{if } t_i \leq t < t_i + \varepsilon \\
\omega_f & \text{if } t_i + \varepsilon \leq t \leq t_f 
\end{cases} \quad (9.3)$$

where $\varepsilon$ is an arbitrary real number satisfying $0 \leq \varepsilon < T$, generally chosen small compared to $T$.

We assume that the motor is endowed with an angular speed sensor (tachymeter). To track the step reference (9.3), the following Proportional-Integral-Derivative (PID) output feedback is used:

$$U = U^* - K_P (\omega - \omega^*) - K_D \dot{\omega} - K_I \int_{t_i}^{t} (\omega(\tau) - \omega^*(\tau)) d\tau \quad (9.4)$$