Finite Element Method and Optimal Control Theory for Path Planning of Elastic Manipulators

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Abstract. Planning of robot trajectory is a very complex task that plays a crucial role in design and application of robots in task space. This paper is concerned with path planning of flexible robot arms for a given two-end-point task in point-to-point motion, based on indirect solution of optimal control problem. We employ the finite element method to modeling and deriving the dynamic equations of robot manipulator with flexible link, so in the presence of all nonlinear terms in dynamic equations open loop optimal control approach is a good candidate for generating the path that optimizes the end effector trajectory. Then the Hamiltonian function is formed and the necessary conditions for optimality are derived from the Pontryagin's minimum principle. The obtained equations establish a two point boundary value problem which is solved by numerical techniques. Finally, simulations for a two-link planar manipulator with flexible links are carried out to investigate the efficiency of the presented method. The results illustrate the power and efficiency of the method to overcome the high nonlinearity nature of the problem.

Keywords: Flexible Manipulator, Finite Element, Optimal Trajectory, Optimal Control.

1 Introduction

Flexible robot arms have some advantages over rigid ones, such as their capability to assure faster motions and a higher ratio of payload to arm weight. However, due to the flexible nature of the system, their dynamic equations are highly non-linear and complex.

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Wang et al. have solved the optimal control problem with direct method using the B-Spline functions in order to determine the maximum payload of a rigid manipulator [1]. The assumed mode expansion method is used by Sasiadek and Green [2, 3] to derive the dynamic equation of fixed base flexible manipulator. In [4, 5] a formulation based on Iterative Linear Programming (ILP) is presented to determine the Maximum Allowable Dynamic Load (MADL) of flexible manipulators. Indeed, the linearizing procedure and its convergence to the proper answer is a challenging issue, especially when nonlinear terms are large and fluctuating, e.g. in problems with consideration of flexibility in links or having high speed motion. As a result, in none of the previous mentioned work which is based on the ILP method, the link flexibility has not been considered either in the dynamic equation or simulation procedure.

None of these published works have used Finite Element Method (FEM) to model and analysis for their systems. One of the main advantages of FEM over the most of other approximate solution methods to modeling the flexible links is the fact that in FEM the connection are supposed to be clamp-free with minimum two mode shape per link. Another significant advantage of FEM, especially over analytical solution techniques is the ease with which nonlinear conditions can be handled. The finite element method has been used to solve very complex structural engineering problems during the past years. The maximum payload of flexible mobile manipulator is determined along the given trajectory by using the finite element approach in [6], so finding the optimal path is not considered in it.

Optimal control can be used in both open loop and close-loop strategies. However, because of the off-line nature of the open loop optimal control in spite of the close-loop ones, many difficulties such as system nonlinearities and all types of constraints may be catered for and implemented easily, so it generally used in analyzing nonlinear systems such as trajectory optimization of different types of robots [7, 8]. It solved by direct and indirect approaches. But, since direct method leads to the approximate solution and this approach is time consuming and quite ineffective due to the large number of parameters involved [9], indirect methods is a good candidate for the cases where the system has a large number of degree of freedom or optimization of the various objectives is targeted [10].

Open-loop optimal control method is proposed as an approach for trajectory optimization of flexible link mobile manipulator for a given two-end-point task in point-to-point motion [11]. But in mentioned paper combined Euler–Lagrange formulation and assumed modes method is used for driving the equation of motions with considering the simply support mode shape and one mode per link. So beside the advantages of this paper over than ILP based ones it can not expressed realistically the behavior of links besides it connection to the motors.

In this paper, for path planning of Elastic manipulators, an indirect solution of the optimal control problem is employed. Dynamic equations are derived using the FEM. Hamiltonian function is formed, and necessary conditions for optimality are obtained from the Pontryagin's minimum principle. These equations establish a Two Point Boundary Value Problem (TPBVP) solved by MATLAB. In comparison with other method the open-loop optimal control method does not require