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Reflexive Analysis of Groups

Vladimir A. Lefebvre

Summary. This chapter develops further a model I previously introduced, of an agent facing a choice between the positive and the negative poles. Here I will consider agents whose individual behavior depends on a 'society' compounded by all of them. Four ideas underlie the theory. The first idea is to consider relationships between the subgroups of agents, not just pairs of agents; this idea allows us to represent a decomposable graph corresponding to an agent or a group of agents as a tree of subgraphs. The second idea is to establish a correspondence between decomposable graphs and polynomials, allowing us to replace a tree of subgraphs with a tree of polynomials representing a computational process. The third idea consists of the interpretation of the tree of polynomials as an agent who has images of the self, which can have images of the self, etc. Finally, the fourth idea is putting an equation into correspondence to the agent, allowing us to find out the agent's state. The theory is illustrated here with several examples from modern geopolitics, including scenarios of current interest.

Introduction

In this work, I develop the ideas described in my book *Algebra of Conscience* (Lefebvre 1982, 2001). I have introduced there a model of an agent facing a choice between the positive and the negative poles. Several predictions of the models have already passed experimental tests (Lefebvre 1980; Lefebvre et al. 1986; Adams-Webber 1997; Grice et al. 2005). Now I will consider agents whose individual behavior depends on a 'society' compounded by all of them.

Imagine a group of agents, each pair of which is either in the relationship of union or that of conflict. Let the group members be involved in work over a certain task and each one has to choose between the active and passive lines of behavior. The active behavior is valued as the positive pole, and the passive behavior as the negative pole. Every agent may experience an influence from the other agents and the source inside the self. In the framework of this scheme, the agent can be in one of four states. In the *first* state, the agent is free to choose any line of behavior, active or passive, depending on circumstances. In

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this state, the agent is able to realize his strategic thinking. In the *second* state, the agent is deprived of the freedom of choice and always chooses the active line of behavior, even if the passive behavior could be more advantageous. In the *third* state, the agent is also deprived of the freedom of choice, but always chooses the passive line of behavior even when it is harmful. In the *fourth* state, the agent is not capable of making a choice at all: it is either inactive or rushing about between two lines of behavior.

The theory described in this work will answer: How to find the states of the agents by knowing the structure of agents' relationships and their influences on one another.

Four ideas underlie the theory. The essence of the first idea is to transfer 'the relationships between the agents' to 'the relationships between the subgroups of agents,' if the agents of one subgroup have the same relationship with the agents of the other subgroup. This idea allows us to represent a decomposable graph corresponding to an agent or a group of agents as a tree of subgraphs.

The second idea consists of establishing a correspondence between decomposable graphs and polynomials. This allows us to replace a tree of subgraphs with a tree of polynomials representing a computational process.

The third idea consists of the interpretation of the tree of polynomials as an agent who has images of the self, which can have images of the self, etc.

Finally, the fourth idea is putting an equation into correspondence to the agent, and this equation allows us to find out the agent's state.

Completed graphs

We presume that the reader is knowledgeable about general definitions in graph theory. Further, we will consider only completed and elementary graphs. A graph is called *completed* if any two nodes a and b are connected by a link (a, b). Links (a, b) and (b, a) are equivalent. A graph is called *elementary* if it consists of one node. We divide a set of all links of a non-elementary graph into two disjoint subsets (one of them can be empty) and call them *relations* R and $\bar{R}$. If $(a, b) \in R$, we say that a and b are connected by link R, which is recorded as aRb. If $(a, b) \in \bar{R}$ then a and b are connected by $\bar{R}$, which is recorded as $a\bar{R}b$. All further definitions for R hold for $\bar{R}$ as well. If two nodes, a and b, can be connected by a sequence of R-type links, we say that a and b are connected in R. If any two nodes of a graph are connected in R, we say that a graph is connected in R. If every node of a graph A is connected with every node of graph B by link R, we write ARB. If graph G consists of subgraphs which are in relation R two-by-two, we say that graph G is divided to these subgraphs. In this case, we will write $G = A_1RA_2R...RA_n$, where $A_1, A_2, ..., A_n$ are subgraphs. The expression $\{a, b, ...\}$ designates a graph with nodes $a, b...$.