Uncoupled 6-dof Tripods via Group Theory

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Abstract: Using kinematic equivalencies originating from the closure of the product in displacement Lie subgroups, we have synthesized seven 6-dof tripods that allow the actuation of the positioning of a center of spherical motion and the independent actuation of the spherical motion around the center. These two actuations are uncoupled. One of these tripods, 3-PRPP(RR), is truly amazing by fully actuating the inputs closely near the base.

1. Introduction

The uncoupled 6-dof (degrees of freedom) parallel manipulators (PMs) with three identical 6-dof limbs allows the actuation of the positioning of a center of 3-dof spherical motion and the actuation of the spherical motion around the center and these two actuations are mutually independent. Uncoupled actuation of 3-dof orientational and 3-dof translational motions is beneficial for the motion planning and the control of such PMs. Several 6-dof PMs are described in Merlet’s book [1]. In most 6-dof PMs, the position and the orientation of moving platform are coupled [3-5]. As authors know, in the literature, only one architecture proposed in Jin et al. [6] is an uncoupled 6-dof tripod. Each limb includes two passive revolute (R) and two passive prismatic (P) pairs, and a 2-dof actuated pair (C≈PR). An algebraic approach based on the Lie-group properties of the displacement set [7-10] can be used to synthesize systematically the architectures of these tripods. We recall the planar-spherical 5-dof motion generators [7]. Three prismatic pairs par-
allel to a Cartesian frame are added to form three 6-dof limbs and the general uncoupled 6-dof tripods are derived. Seven novel uncoupled tripods are introduced.

2. Planar-Spherical Motion Generators

The spherical pair (S) of center O embodies the group \( \{S(O)\} \) of spherical motions around \( O \) and can be replaced by a serial array of three revolute pairs that have non-coplanar axes intersecting at \( O \), as shown in Fig.1. All these RRR chains generate a group of spherical rotations in a neighborhood of the identity. When three R axes are coplanar, an RRR spherical chain is singular. Special realizations that are propitious to a large singularity-free workspace are obtained when the R axes are 2-by-2 perpendicular at an initial posture. In the paper, we use planar-spherical motion generators with an R that is perpendicular to the plane of the planar-spherical motion. In Fig.1(b), the non-depicted plane is horizontal.

(a) S pair  (b) RRR with vertical first R axis

Fig. 1 Spherical pair and equivalent chain

The planar kinematic pair (F) that is perpendicular to a unit vector \( u \) produces a group \( \{G(u)\} \) of planar gliding displacements. The equivalent generators of planar displacements are depicted in Fig.2 [8], in which all the chains produce a 3D group of planar displacements in a neighborhood of the identity transform, which represents the absence of motion. The singular postures of such chains are described in [9]. The product of a 3-dimensional (3D) planar group and a 3D spherical group is \( \{G(u)\}\{S(O)\} \) with \( \{G(u)\}\cap\{S(O)\}=\{R(O,u)\} \) where \( \{R(O,u)\} \) denotes the group of rotations around the axis \( (O,u) \) passing through \( O \) and parallel to \( u \). A chain embodying \( \{G(u)\}\{S(O)\} \) has the internal mobility represented by \( \{R(O,u)\} \). The elimination of the superfluous square of \( \{R(O,u)\} \) in the product \( \{G(u)\}\{S(O)\} \) leads to two main families of irreducible representations of \( \{G(u)\}\{S(O)\} \), which is called a planar-spherical (\( Pl-Sp \)) motion (or kinematic bond). The \( Pl-(RR) \) limbs constitute one family of 5-dimensional (5D) planar-spherical motion generators [7]; \( Pl \) denotes a 3-dof planar chain, (RR) a 2-dof spherical chain. The \( Pl-(RR) \) chains can be a 5-dof sub-chain of a chain producing 6-dof motion. All serial chains of structural type \( Pl-(RR) \) are enumerated in Fig.3. These chains generate a 5D set of motions containing two 3D groups.

(a) F pair  (b) RRR  (c) RRP  (d) PRR  (e) RPR  (f) PRP  (g) RPP  (h) PPR

Fig. 2 Planar pair and equivalent generators