Non-singular assembly mode change in 3-RPR-parallel manipulators

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Abstract Non singular assembly mode change of parallel manipulators has been discussed for a while within the robotics community. This term means that a parallel robot can pass from one solution of the direct kinematics into another without crossing a singularity. In this paper we will show that opposed to the accepted opinion all general planar 3-RPR parallel manipulators have this ability. Using geometric properties of the singularity surface of this manipulator we will give a rigorous mathematical proof for this proposition. This proof will use the fact that the singularity surface is a fourth order surface having only very special singularities. A secondary result of this proof will be the first proof for the widespread used property that the singularity surface divides the workspace of the manipulator into two aspects that are path connected. We derive a simple technique how to construct singularity free trajectories that join all assembly modes of one connected component.

1 Introduction

The motivation for this paper is to clarify some open problems concerning the properties of non singular assembly mode change of general 3 – RPR-parallel manipulators. This type of parallel mechanism consists of a base and a platform. Three points of the base are connected to the platform via extensible legs mounted to base and platform with revolute joints. The layout of the mechanism and the corresponding coordinate systems are displayed in Fig.1. The kinematics, workspace and singularities of this mechanism were studied in several papers (see for instance [2, 3]).

Recently some authors have brought up again the issue of non singular assembly mode change in parallel manipulators ([10, 12, 14]). The existence of continuous paths joining two solutions of the direct kinematics without crossing a singularity

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is old. The discussion of non-singular assembly mode change started with a paper by Innocenti and Parenti-Castelli [11]. In the following years several authors tried to link this property with the notion of cuspidality (see esp. [10] and the references there). But there are doubts to some assumptions that have been used in almost all of the papers without proof. The most questionable among these assumptions is that the singularity surface divides the workspace into two aspects (i.e. into two disjoint path-connected components). It will turn out that this assumption is correct but the proof is not trivial. It needs a generalization of the famous Jordan curve theorem to surfaces in 3d space. In this paper the rational parametrization of the singularity surface \( S \) developed in [6] will be used to obtain an analytic expression of its counterpart \( \bar{S} \) in the joint space. It turns out that \( \bar{S} \) is algebraic and of degree 12. With the analytic expression it is possible to get a more accurate representation of \( \bar{S} \) than those published in [10] or [12]. The algebraic representation of \( S \) in the kinematic image space will allow to proof that every general 3 – RPR planar parallel manipulator has the property of non-singular assembly mode change.

The paper is organized as follows: In Section 2 we briefly recall the kinematic mapping, the constraint equations and their use to solve the direct kinematics of the 3 – RPR manipulator. Furthermore we discuss the singularity surface \( S \) in the image space and its rational representation. In Section 3 we use the rational representation to map \( S \) to the joint space. In Section 4 we use all the collected information to proof the non-singular assembly mode change property and show in a general example how simple assembly mode changing trajectories can be constructed.

2 Direct kinematics and the singularity surface in the kinematic image space

A Euclidean displacement \( \mathcal{D} \) of a plane \( \Sigma \) can be written as \( p_0 = A \cdot p \), where \( p \) is a vector whose entries are the homogeneous coordinates of the moving point \( P(1 : px : py) \), \( p = (1, px, py)^T \), expressed in \( \Sigma \). \( p_0 \) represents the same point in the fixed system \( \Sigma_0 \). The matrix \( A \) is given by