

## Chapter 9

# Exploratory Analysis of Stochastic Local Search Algorithms in Biobjective Optimization

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**Abstract** This chapter introduces two Perl programs that implement graphical tools for exploring the performance of stochastic local search algorithms for biobjective optimization problems. These tools are based on the concept of the empirical attainment function (EAF), which describes the probabilistic distribution of the outcomes obtained by a stochastic algorithm in the objective space. In particular, we consider the visualization of attainment surfaces and differences between the first-order EAFs of the outcomes of two algorithms. This visualization allows us to identify certain algorithmic behaviors in a graphical way. We explain the use of these visualization tools and illustrate them with examples arising from practice.

## 9.1 Introduction

Experiments in computer science often produce large amounts of data, mainly because experiments can be set up, performed, and repeated with relative facility. Given the amount of data, exploratory data analysis techniques are one of the most important tools that computer scientists may use to support their findings. In particular, specialized graphical techniques for representing data are often used to perceive trends and patterns in the data. For instance, there exist techniques for the extraction of relevant variables, the discovery of hidden structures, and the detection of outliers and other anomalies. Such exploratory techniques are mainly used during the design of an algorithm and when comparing the performance of various algorithms.

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Even before testing more formal hypotheses, the algorithm designer has to find patterns in experimental data that provide further insights into new ways of improving performance.

In this chapter, we focus on the graphical interpretation of the quality of the outcomes returned by stochastic local search (SLS) algorithms (Hoos and Stützle 2005) for biobjective combinatorial optimization problems in terms of Pareto optimality. This notion of optimality is tied to the notion of dominance. We say that a solution dominates another one if it is at least as good as the latter for every objective and strictly better for at least one objective. For these problems, the goal is to find the set of *nondominated* solutions among all feasible solutions. The mapping of these solutions in the objective space is called the *Pareto-optimal front*. For the particular case of multiobjective combinatorial optimization problems (MCOPs), fundamental results about their properties and complexity are given by Ehrgott (2000).

Usually, each run of an SLS algorithm produces a nondominated set, a random set of mutually nondominated objective vectors that approximates the Pareto-optimal front of an MCOP. Currently, there are two widely used techniques for assessing the performance of these algorithms with respect to the solution quality: graphical examination of multiple outcomes and scalar quality indicators. Unfortunately, these two approaches present several drawbacks that have been discussed in the literature (Knowles and Corne 2002, Zitzler et al. 2003).

The empirical attainment function (EAF), formally described in Chap. 5, is seen in this chapter as a middle ground between directly plotting the complete output and the extreme simplification of quality indicators. The EAF is a summary of the outcomes of multiple runs of an SLS algorithm, and, at the same time, it is sufficiently complex to detect whether and where an algorithm is better than another. By plotting and comparing the EAFs of different SLS algorithms, we are able to pinpoint several performance behaviors that otherwise would be hidden when using other performance assessment approaches.

The chapter is organized as follows. Section 9.2 provides a basic introduction to SLS algorithms for multiobjective optimization in terms of Pareto optimality, and their performance assessment. Sections 9.3 and 9.4 introduce plotting techniques for exploring algorithm performance based on the EAF, and describe two Perl programs, `eafplot.pl` and `eafdiff.pl`. Section 9.5 presents three examples of applications of these programs. Finally, we present conclusions and further work in Sect. 9.6.

## 9.2 Stochastic Local Search for Multiobjective Problems

SLS algorithms iteratively search for good quality solutions using the local knowledge provided by the definition of a neighborhood or a set of partial solutions. Since they are based on a randomized search process, it is not expected that the same outcome is returned for different runs with different random seeds of the random number generator. *Metaheuristics* are general-purpose SLS methods that can be adapted