Chapter 4
Robust SMC for Uncertain Time-Delay Systems

4.1 Introduction

The control problem of time-delay systems has received considerable attention over the past years, and different design approaches have been proposed in [24, 128, 89, 31, 39, 136]. However, they are sensitive to the uncertainty, which directly affects the control systems.

An alternative approach is sliding mode control (SMC), which has many attractive features such as a fast response with asymptotic stability. The salient advantages of this method are: (i) when the state is constrained to the sliding surface, SMC can completely reject uncertainties which satisfy the matching condition; and (ii) the high possibility of stabilizing some complex non-linear systems which are difficult to stabilize by state feedback laws. Because of these advantages, variable structure control theory has found applications to various kinds of plants ([85]). The existence condition of a linear sliding surface for systems with mismatched uncertainties was given in [25, 26, 100], but their methods cannot be applied to systems with time-delay. In [161], a new robust stability criterion for uncertain time-delay systems is given and the SMC is proved to be applicable. There, due to use of matrix norm, the result is more or less conservative and complicated which leads to inconvenience in designing the sliding surface for uncertain time-delay systems.

In this chapter, we consider how to design sliding surface and reaching motion controller for a class of time-delay systems with mismatched uncertainties and matched exogenous disturbance. An LMI condition for the existence of linear sliding surfaces is derived. The solution to the condition can be used to characterize linear sliding surfaces, and by selecting suitable reaching law the reaching motion controller is designed. Our methods have the advantages in computation since the given stability condition is represented by the LMI which can be very efficiently solved by using powerful LMI algorithm [15]. Finally, we extend our results to the interval systems with time-delay.
4.2 Problem Formulation

Consider the uncertain time-delay system of the form

\[
\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - \tau)
+ B(u(t) + Fw(t))
\]
\[x(t) = \varphi(t), t \in [-\tau, 0] \tag{4.1}\]

where \(x(t) \in R^n\) is the state, \(w(t) \in R^l\) is the disturbance whose each component is bounded by the known \(\bar{w}_i(t)\), i.e., \(w_i(t) \leq \bar{w}_i(t), i = 1, 2, \cdots, l\), \(u(t) \in R^m\) is the control input, \(A, A_d, B\) and \(F\) are real constant matrices with appropriate dimensions and \(\text{rank}(B) = m\). The model uncertainties are described by

\[
\Delta A = \sum_{i=1}^{p} \alpha_i(t)A_i, \quad |\alpha_i(t)| \leq 1
\]
\[\Delta A_d = \sum_{i=1}^{q} \beta_i(t)A_{di}, \quad |\beta_i(t)| \leq 1 \tag{4.2}\]

where the matrices \(A_i, i = 1, \cdots, p\) and \(A_{dj}, j = 1, \cdots, q\) are known with \(\text{rank}(A_i) = a_i\) and \(\text{rank}(A_{dj}) = a_{di}\), \(\alpha_i(t)\) and \(\beta_i(t)\) are Lebesgue-measurable. Suppose \(A_i\) and \(A_{di}\) have full rank factorization of \(A_i = G_iH_i\) and \(A_{di} = G_{di}H_{di}\), respectively, where \(G_i \in R^{n \times a_i}, H_i \in R^{a_i \times n}, G_{di} \in R^{n \times a_{di}}\) and \(H_{di} \in R^{a_{di} \times n}\). Then

\[
\Delta A = \sum_{i=1}^{p} \alpha_i(t)A_i = \left[ G_1 \ G_2 \ \cdots \ G_p \right] D \left[ H_1 \ H_2 \ \cdots \ H_p \right]^T = G D H \tag{4.3}
\]
\[\Delta A_d = \sum_{i=1}^{q} \beta_i(t)A_{di} = \left[ G_{d_1} \ G_{d_2} \ \cdots \ G_{d_q} \right] D_d \left[ H_{d_1} \ H_{d_2} \ \cdots \ H_{d_q} \right]^T = G_d D_d H_d
\]

where

\[
D = \text{diag} \left( \alpha_1(t)I_{a_1 \times a_1} \ \cdots \ \alpha_p(t)I_{a_p \times a_p} \right)
\]
\[D_d = \text{diag} \left( \beta_1(t)I_{a_{d1} \times a_{d1}} \ \cdots \ \beta_q(t)I_{a_{dq} \times a_{dq}} \right) \tag{4.4}\]

According to the structure of \(D\) and \(D_d\), the following scaling matrices are defined as

\[
S_D = \{ Y | Y = \text{diag} \left( Y_1 \ Y_2 \ \cdots \ Y_p \right), 0 < Y_i = Y_i^T \in R^{a_i \times a_i} \}
\]
\[S_{D_d} = \{ Y_d | Y_d = \text{diag} \left( Y_{d1} \ Y_{d2} \ \cdots \ Y_{dq} \right), 0 < Y_{di} = Y_{di}^T \in R^{a_{di} \times a_{di}} \} \tag{4.5}\]