6 Means and Means of Means

We shall confine ourselves to unweighted and weighted arithmetic means as issued by the method of least squares.

6.1 Arithmetic Mean

Let us consider \( n \) independent repeated measurements

\[ x_1, x_2, \ldots, x_n \]

with formal decompositions

\[ x_l = x_0 + (x_l - \mu_x) + f_x, \quad -f_{s,x} \leq f_x \leq f_{s,x}; \quad l = 1, \ldots, n. \]

The measurements produce a mean

\[ \bar{x} = \frac{1}{n} \sum_{l=1}^{n} x_l \quad (6.1) \]

and an empirical standard deviation

\[ s_x^2 = \frac{1}{n-1} \sum_{l=1}^{n} (x_l - \bar{x})^2. \quad (6.2) \]

As shown in Appendix H, Student’s \( t \) exists in two versions

\[ T(n-1) = \frac{X - \mu_x}{S_x} \quad \text{and} \quad T(n-1) = \frac{\bar{X} - \mu_x}{S_x/\sqrt{n}}. \]

Correspondingly, the inequality

\[ -t_P \leq T \leq t_P \]

issues confidence intervals

\[ x_l - t_P(n-1) s_x \leq \mu_x \leq x_l + t_P(n-1) s_x; \quad l = 1, \ldots, n \quad (6.3) \]
Fig. 6.1. Biased arithmetic mean $\bar{x}$ with uncertainty $u_{\bar{x}}$