Chapter 4
An Improvement in Performance of Input-Delay System Using Nonlinear Sliding Surface

4.1 Introduction

It is well recognized that the existence of a time delay may affect the performance or result in loss of stability as shown in [65]. Recently many methods have been published in [105, 104, 57, 83, 103] on the design of control laws for time-delay systems; see the references therein. In [77], it has been shown that all the proposed methods for time delay systems use prediction of state either explicitly or implicitly. Control algorithms based on state prediction were first proposed in [72]. Furukawa and Shimemura [43] proposed a predictor-observer based scheme to control plants with time-delay. Using the predictor, the original input-delay system can be converted into a delay-free system and the problem reduces to finite dimensions. The predictor is used when the delay is known which restricts its scope. However, Lozano et al. [71] consider uncertainty in the knowledge of delay with a state predictor based scheme. Recently, in [104], sliding mode control is proposed based on a discrete predictor for a regulator case. In [71], a state predictor based state feedback control law is proposed; and the authors also propose a predictor in the discrete-time framework. The performance of a system is adversely affected by a delay in the input as shown in [65, 83], which necessitates compensation. In this chapter, it has been shown that how the performance of input-delay systems can be improved by using a nonlinear sliding surface unlike the use of a linear sliding surface (linear in the predicted states). A nonlinear sliding surface is designed in predicted state. Furthermore, it has been shown if performance of the system transformed in the predicted state is improved then it leads to the improvement of the performance of the original time-delay system. The general uncertain system is considered which contains both matched and unmatched perturbations. It is an established fact that for an uncertain system, discrete-time sliding mode is possible only in the vicinity of sliding surface $s(k) = 0$. To ensure ideal sliding motion $s(k) = 0$ for an uncertain system, the exact value of disturbance/uncertainty is needed. In general, the exact value of disturbance/uncertainty is not known. Therefore, in this chapter an ultimate boundedness of resulting motion is proved. This chapter extends the results of the previous chapter for a system with a delay and having both matched and unmatched uncertainties. This chapter is based on authors work in [7].
A brief outline of the chapter is as follows. The discrete-time state predictor is given in Section 2. Section 3 contains the structure of the nonlinear sliding surface. Section 4 presents the control law. Stability of motion and ultimate boundedness is proved in Section 5. An example and simulation results are presented in Section 6 followed by the conclusions in Section 7.

4.2 Discrete-Time Predictor

In this section, we review the concept of the discrete-time state predictor as proposed in [71, 104] for the system

\begin{align*}
    x(k+1) &= \Phi x(k) + \Gamma u(k-h) + D \rho(k), \\
    y(k) &= C_1 x(k), \\
    u(k) &= \Theta(k) \quad k = -h, -h+1, \ldots 0
\end{align*}

where \( x(k) \in \mathbb{R}^n, \ u(k) \in \mathbb{R}, \ y(k) \in \mathbb{R} \) are respectively the state, the input, and the output of the system. \( \Phi, \Gamma, C_1 \) are matrices of appropriate dimensions, \( h \) is an integer which denotes the amount of delay and \( \Theta(k) \) denotes the initial condition. \( \Theta(k) \) is generally available because it refers to past inputs which were applied to the system in past. \( D \) is a column matrix and \( \rho(k) \in \mathbb{R} \). \( D \rho(k) \) accounts for uncertainty which contains both matched and unmatched components.

**Remark 4.2.1.** It is assumed that the system matrix \( \Phi \) is non-singular. When \( \Phi \) is singular then a possible way to circumvent this problem would be to slightly perturb the elements of \( \Phi \) for ensuring non-singularity. Appropriate sampling time can also be chosen to ensure non-singularity of \( \Phi \) when discrete-time model is obtained from its continuous time counterpart.

It is also assumed that the pair \((\Phi, \Gamma)\) is controllable. The predictor can be used to convert the system (4.1) into a system which does not explicitly contains delay. A control input applied at the \( k^{th} \) sampling instant becomes effective at the \((k+h)^{th}\) sampling instant due to the delay in the input. This situation demands that at the \( k^{th} \) instant the controller should know the future value of the state at the \((k+h)^{th}\) instant. This can be accomplished by predicting the state from the plant dynamics.

Consider the deterministic part of (4.1) (i.e. \( \rho(k) = 0 \)), then

\begin{align*}
    x(k+2) &= \Phi x(k+1) + \Gamma u(k-h+1), \\
    x(k+3) &= \Phi x(k+2) + \Gamma u(k-h+2), \\
    x(k+3) &= \Phi^3 x(k) + \Phi^2 \Gamma u(k-h) + \\
    &\quad \Phi \Gamma u(k-h+1) + \Gamma u(k-h+2), \\
    &\quad \vdots \\
    &\quad \vdots \\
    x(k+h) &= \hat{x}(k) = \Phi^h x(k) + \sum_{i=-h+1}^{0} \Phi^{-i} \Gamma u(k+i-1). \quad (4.2)
\end{align*}