Chapter 6
Multi-objective Sliding Mode Design Using Full-Order Lyapunov Matrix

6.1 Introduction

In this chapter, the sliding mode control (SMC) design for a class of continuous-time linear uncertain systems is considered based on the parametric approaches utilizing the Lyapunov (or Riccati-like) inequalities. SMC for the continuous-time linear systems is one of the well-known issues in control theory. However, this chapter addresses a new aspect of SMC for linear uncertain systems and establishes a systematic procedure to design a sliding hyperplane having multiple design objectives.

In the literature, much effort has been made to design sliding modes that satisfy the desired performance criteria. The well-known criteria include quadratic performance optimization [101], guaranteed $H_2$ cost minimization [90], eigen-structure assignment including pole-clustering [25, 31, 37], robustness to parametric uncertainties [88, 64, 107], and so on. Note that all these approaches are concerned with satisfying a single design objective. Moreover, the design objectives have not been presented in a unified framework.

On the other hand, remarkable progress has been made in linear control theory for solving the optimal problems with multiple constraints based on the LMIs (e.g., see [24] and [84]). The basic idea of the multi-objective approach based on LMIs is to seek a common Lyapunov matrix that simultaneously satisfies different parametric constraints imposed by the design performances. Assuming that the common variables may cause the conservatism, however, it does provide the flexibility of the control design with multiple objectives and the ease of synthesis in the parameter space based on the LMIs [17]. Also, there have been notable results in reducing the design conservatism in the literature (e.g., see [60] and [86]).

This chapter reconstructs the research results proposed in [61, 64, 62] and [63] in a unified LMIs framework. The proposed approach effectively solves the constrained optimization problems by adopting the common Lyapunov matrix idea of the linear control theory. First, through Sections 6.2 and 6.3 the sliding hyperplane design is considered for a class of uncertain systems with parametric uncertainties that are either matched or mismatched. Regardless of the matching condition of the model uncertainties, the quadratic stabilizability will be shown to
be a sufficient condition for the existence of stable sliding modes. This enables the definition of a guaranteed cost optimization on the sliding hyperplane. Then, in Section 6.4 attention is paid to the quadratic performance optimization problem with the pole-clustering constraint in sliding mode. To this end, the parametric constraints for quadratic performance and pole-clustering requirement are derived in the forms of LMIs that are typically devised with the proposed parameterization technique, leading to the multi-objective optimization framework. Finally, in Section 6.5 the conclusion follows.

6.2 Parameterization of Sliding Mode Using the Lyapunov Matrix

6.2.1 System Description: Nominal Case

Consider the system

\[ \dot{x}(t) = Ax(t) + Bu(t) + Fw(t), \]  

where \( x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \) are the state vector and the control input, respectively, and \( w \in \mathbb{R}^l \) is a disturbance in which each element is bounded as \( |w_j(t)| \leq \overline{w}_j, \forall j \in [0, l] \) for the known \( \overline{w}_j \)'s. Suppose that the system is of the regular form \[101, 30\]

\[
\begin{align*}
\dot{x}_1 &= A_{11}x_1 + A_{12}x_2 \\
\dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + B_2u + F_2w,
\end{align*}
\]

where \( x_1 \in \mathbb{R}^{n-m} \), \( x_2 \in \mathbb{R}^m \), \( B_2 \in \mathbb{R}^{m \times m} \) nonsingular and \( F_2 \in \mathbb{R}^{m \times l} \). Observe that the matching condition is assumed for the external disturbance, \( w(t) \). Without loss of generality, it can be seen that a similarity transformation always exists that converts the original system in a physical coordinate into the regular form system, as in (6.2). Also, let the stabilizability of the pair \((A, B)\) be assumed.

Consider the switching function

\[ s(t) = c_1x_1 + x_2 \]

for some \( c_1 \in \mathbb{R}^{m \times (n-m)} \). Suppose that a control law is employed to satisfy the reachability condition such that \( \dot{s}(t)^T s(t) < 0, \forall t > 0 \), which achieves the sliding mode such that

\[ s(t) = 0, \forall t \geq t_s \]

for some \( t_s > 0 \). In this chapter, let the control law be given by

\[ u = \begin{cases} 
0, & (\|s(t)\| = 0) \\
-B_2^{-1} \left\{ c^T A x + \beta s + Z(t) \text{sign}(s) \right\}, & (\|s(t)\| > 0),
\end{cases} \]

where \( \text{sign}(s) = [\text{sign}(s_1), \ldots, \text{sign}(s_m)]^T \), \( \beta > 0 \), \( c^T = [c_1, I_m] \), and \( Z(t) = \text{diag} [z_1, \cdots, z_m] \) for \( z_i \) defined as