On the Robustness of Type-1 and Type-2 Fuzzy Tests vs. ANOVA Tests on Means

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Abstract. This paper presents a simulation study on fuzzy tests vs. ANOVA test on means. Type-1, Interval Type-2 and ANOVA classical tests are compared through a simulated experiment for contrasting the stability of those approaches in front to a small change on sample.

We perform an experiment of comparing the means of three groups where the classical ANOVA test is very nearby to the rejection p-value and the fuzzy tests get more robust results. In this way, we use bootstrap concepts to simulate the change of a random value of the sample to view the behavior of each technique in front to these changes.

1 Introduction and Motivation

Recently, the use of fuzzy sets for involving uncertainty in the statistical analysis and its advantages has allowed the appearance of a new discipline called Fuzzy Statistics, in which many researchers are dedicating their efforts to the definition of correct expressions for solving different problems of data analysis. James J. Buckley in [1] and [2] defines new probability concepts based on Type-1 fuzzy sets called “Fuzzy Probabilities” and therefore fuzzy test statistics. A. Mohammadpuor & A. Mahammad.Djafari in [3] propose a fuzzy test by using fuzzy relations and Bayesian concepts, showing their proposal converges to the best crisp test by using the Neyman-Pearson Lemma. B. F. Arnold in [7] proposes a similar work based on interval numbers and M. Last, A. Schenker & A. Kandel in [8] show an application to medical data.

We compare three hypothesis engines: ANOVA test, a Type-1 fuzzy logic system (T1FLS) proposed by Figueroa & Soriano in [9] and an Interval Type-2 fuzzy logic system (IT2FLS) proposed by Figueroa, Rodriguez & Sierra in [10] in order to view its behavior when the sample has been changed.

1 For additional information see [4], [5] and [6].
This paper is divided as follows: Section 1 introduces the topic; Section 2 presents hypothesis testing; Section 3 describes the two fuzzy logic test systems; in Section 4 we present the methodology of simulation; Section 5 presents the results of the simulation and Section 6 gives some concluding remarks.

2 Hypothesis Testing

In many hypothesis-testing problems two hypothesis are discussed: The first, the hypothesis being tested, is called the Null Hypothesis, denoted by $H_0$, and the second is called the Alternative Hypothesis, denoted by $H_a$. The natural supposition is that if $H_0$ is false, then $H_a$ is true. The $H_0$ outlined here is:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_n \quad (1)$$

$H_a : \text{At least one mean is different from others.}$

The multiple means case (1) is widely treated by using the ANOVA method. The Table 1 shows a brief description of the test.

<table>
<thead>
<tr>
<th>Null Hypothesis Test Statistic</th>
<th>Alternative Hypothesis Reject Criterion</th>
</tr>
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<tbody>
<tr>
<td>$H_0 : \mu_1 = \mu_2 = \cdots = \mu_n$</td>
<td>Reject $H_0$ when $F_0 \leq F_{1-a, a-1, N-a}$</td>
</tr>
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</table>

Where $a$ is the Amount of Data groups, $n_i$ is the Number of observations of the $i_{th}$ group, $N$ is the Total number of observations, $MS_{means}$ is the Mean Square of means, $MS_E$ is the Mean Square of error, $y_{ij}$ is the $j_{th}$ observation for the $i_{th}$ data group, $j = 1, 2, \cdots, n$, $\bar{y}_i$ is the Mean of the $i_{th}$ data group, $i = 1, 2, \cdots, a$ and $\bar{y}_.$ is the Mean of the complete data set. For more information see Scheffé in [11] and Searle in [12].

2.1 Decision Making

A crisp bilateral hypothesis test is a simple process that either rejects or accepts $H_0$ by using a pre-defined $\alpha$ confidence level (Usually $\alpha = 0.05$), assuming it as the correct one, based on any asymptotic test statistic. The usual reasoning is to Accept $H_0$ if the sample statistic is inside a Confidence Interval, that is:

$$\bar{y} \in \left[ g_1(\mu_0) ; g_2(\mu_0) \right] \quad (2)$$

Where $g_1(\mu_0)$ and $g_1(\mu_0)$ are functions of $\mu_0$ and $\bar{y}$ is the mean of the sample $y$. $P(g_1(\mu_0) < \mu < g_2(\mu_0)) = 1 - \alpha$. The classical hypothesis tests use only one rule to verify $H_0$, based on asymptotic properties (Or Normality) of the sample:

$R_1$: If $\bar{y} \in \left[ g_1(\mu_0) ; g_2(\mu_0) \right]$ then $H_0$ is Accepted, otherwise $H_0$ is Rejected.