Chapter 13
Combining Market and Credit Risk

The valuation approach detailed in Chap. 4 is centered on estimating the distribution of future values of a transaction after having simulated trajectories of the underlying stochastic drivers. When markets are complete, the pricing-by-arbitrage paradigm allows us to price stochastic payoffs as an expectation in a particular measure, namely the one under which the prices of assets are martingales when expressed in units of a chosen numeraire.

In the context of Chap. 4, this means performing the AMC estimation algorithm on simulations drawn from this martingale measure, which we have denoted throughout by $\text{N}$. In practice, one needs to know the probability distribution of future trade values also in measures other than that under which the simulation takes place. For example:

(i) Asset price processes evolve in the real-world measure, and not the risk-neutral one used for pricing.

(ii) In assessing market risk, one would look at the value distribution under a measure where chosen underlying risk factors are conditioned to have a chosen behaviour, e.g. under a stress scenario.

(iii) In measuring counterparty credit risk, issues of right-way/wrong-way risk arise. This happens when the value of a transaction and the quality of the counterparty are not independent. In this case, conditioning on the event of counterparty default alters the value distribution, so that credit exposure should be analysed in a measure different to that used for pricing.

All the above problems reduce to analysing the value distribution in a probability measure different to that used for simulation. Importantly, we would like such analysis to be possible without having to perform additional simulations, and we show in this chapter how this can be achieved using well-known change-of-measure techniques. In particular, we present and compare two approaches to right-way/wrong-way risk issues, a topic of central importance from a regulatory perspective.\(^1\)

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\(^1\) Under Basel II right-way/wrong-way exposures have to be identified and computed accordingly.

13.1 Change of Measure: Practical Implementation

In Appendix B we describe the classical mathematical set-up and notation to change measure. In this section we highlight how these techniques can be applied in practice within our framework.

Our AMC estimation algorithm results in samples of values of the distribution of future values of the transaction under the measure \( \mathbb{N} \), with, say, \( \hat{F}_t^\mathbb{N} \), being the time-\( t \) empirical distribution under \( \mathbb{N} \), obtained as

\[
\hat{F}_t^\mathbb{N} (v) = \mathbb{P} \left[ V_t < v \right] = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{V_t(i) < v}, \quad v \in \mathbb{R};
\]

(13.1)

here, \( V_t \) is the distribution of time-\( t \) prices (which has been obtained in \( \mathbb{N} \)). The \( \hat{\cdot} \) indicates quantities obtained empirically from sample distributions.

The goal is now to use the change of measure technique to obtain \( \hat{F}_t^\mathbb{P} \), the empirical distribution under an alternative measure \( \mathbb{P} \). For this purpose we can use the Radon-Nikodym derivative

\[
\zeta_t = \frac{d\mathbb{P}}{d\mathbb{N}} \bigg|_{\mathcal{F}_t}
\]

(13.2)

to write, for each \( v \in \mathbb{R} \),

\[
F_t^\mathbb{P} (v) := \mathbb{P}[V_t \leq v] = \mathbb{E}^\mathbb{P} [\mathbb{1}\{V_t \leq v\}] = \mathbb{E}[\mathbb{1}\{V_t \leq v\} \zeta_t].
\]

(13.3)

Of course, expression (13.3) will need to be computed empirically from the sample of values \( \{V_t(i)\} \) and the corresponding values \( \{\zeta_t(i)\} \), resulting in the empirical version, \( \hat{F}_t^\mathbb{P} \), of \( F_t^\mathbb{P} \), by replacing expectations with sample means.

Using the empirical estimates of the CDF’s \( \hat{F}_t^\mathbb{P} \) and \( \hat{F}_t^\mathbb{N} \), we obtain a sample of \( V_t \) under \( \mathbb{P} \) by setting

\[
\tilde{V}_t(i) = \left( \hat{F}_t^\mathbb{P} \right)^{-1} \left( \hat{F}_t^\mathbb{N} \left( V_t(i) \right) \right),
\]

(13.4)

where the superscript \( (i) \) indicates the \( i \)’th of a sample of values, and where the \( \tilde{\cdot} \) signifies values of \( V_t \) under the changed measure \( \mathbb{P} \). From the sample \( \{\tilde{V}_t(i)\} \), risk measures such empirical quantiles under the measure \( \mathbb{P} \) can now be computed in the same way as for the original sample of values \( \{V_t(i)\} \).

To illustrate with a particular example, consider having a transaction driven by \( d \) risk factors, and suppose we want to compute exposure in the real-world measure. Let \( X \) be standard Brownian Motion in \( \mathbb{R}^d \), and let \( Y \equiv RX \) (with \( R^T R \) positive

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