In Chapter 8 the QL theory of macroscopic games was developed on the basis of quantum logic. These investigations were initiated by Andrey Grib and then continued in collaboration with me. They induced understanding that games for macroscopic players having all the distinguishing features of “really quantum games” (i.e., games that are based on microscopic sources of randomness such as pairs of entangled photons) can be easily constructed and represented by means of quantum logic. This QL (in fact, quantum logic) program on macroscopic games stimulated the author to apply the contextual statistical model to represent nonclassical (from the probabilistic point of view) games in the contextual form and then, using QLRA, map them into the complex (or even hyperbolic) Hilbert space. We shall do this in the present chapter. One of the consequences of considerations in this chapter is that in order to simulate QL games one does not need sources of quantum particles, e.g., photons. It is enough to use classical random generators.

9.1 Quantum Probability and Game Theory

We present a simple game between macroscopic players, say Alice and Bob (or in a more complex form - Alice, Bob and Cecilia), which can be represented in the QL way – by using a complex probability amplitude (game’s “wave function”) and non-commutative operators. The crucial point is that the games under consideration are so-called extensive form games, see e.g. [70]. Here the order of actions of players is important; such a game can be represented by a tree of actions. The QL probabilistic behavior of players is a consequence of incomplete information, which is available to e.g. Bob about the previous action of Alice. In general one cannot construct a classical probability space underlying a QL game – even for a QL game with two players. In a QL game with three players Bell’s inequality for averages of payoffs can be written. It can be violated. The most natural probabilistic description in such a framework is given by our contextual probability theory. We shall use QLRA to find QL representations of extensive form games. The probabilistic structure of our game (for two players) can be considered as Gudder’s probability manifold [126, 127] with the atlas having two charts.
The QL behavior can be produced by local gambling. However, QL gambling can be completed by interactions between players (of course, laws of special relativity are not violated).

### 9.2 Wine Testing Game

A restaurant has a good collection of (only) French and Italian wines of various sorts. Couples come to this restaurant for dinner, and to have more fun they play the following Wine Game, which consists of two wine tests.

A1) Alice selects a bottle (without telling her friend Bob the wine’s name) and proposes that he tests the wine. A bottle of this wine is opened in the restaurant’s kitchen; Bob gets just a glass of this wine. Alice asks him the question:

“Is it French or Italian?”

A2) If Bob answers (after testing) correctly, he gets some amount of money; if not, he loses money and Alice gets some amount of money.

The choice in A1 is not totally random, Alice has her own preferences (later she wants to share the chosen bottle with Bob).

In the second part of the game Alice and Bob interchange their roles, so Bob starts by choosing a bottle of French or Italian wine and so on.

We introduce for the first and second parts of the game the elements of the payment matrices

\[
(h_b^{FF;1}, h_b^{FI;1}, \ldots), \quad (h_a^{FF;1}, h_a^{FI;1}, \ldots), \quad k = 1, 2.
\]

Here the indexes \( k = 1, 2 \) denote the first and second part of the game and \( FI, \ldots, II \) combinations of choices of Alice and Bob.\(^1\) The upper indexes \( a, b \) are marks for Alice’s and Bob’s payoffs. It is natural to assume that

\[
h_b^{FF;1}, h_b^{II;1} > 0, \quad h_b^{FI;1}, h_b^{IF;1} < 0
\]

as well as

\[
h_a^{FI;1}, h_a^{IF;1} > 0, \quad h_a^{FF;1}, h_a^{II;1} < 0.
\]

In the zero sum game

\[
h_b^{FF;k} = -h_b^{FF;k}, \quad \ldots, \quad h_b^{II;k} = -h_b^{II;k}.
\]

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\(^1\) We also remark the Alice’s choice can be considered as an “element of reality”, since her, e.g., \( F \) is really French wine, but Bob’s \( F \) may be in reality either French or Italian wine, cf. with discussions about realism in quantum mechanics, e.g., [87, 88, 277].