Chapter 4
Monetary and Fiscal Cooperation between Europe and America

1. The Model

The model of unemployment, inflation, and the structural deficit can be characterized by a system of six equations:

\[ u_1 = A_1 - M_1 + 0.5M_2 - G_1 - 0.5G_2 \]  \hspace{1cm} (1)
\[ u_2 = A_2 - M_2 + 0.5M_1 - G_2 - 0.5G_1 \]  \hspace{1cm} (2)
\[ \pi_1 = B_1 + M_1 - 0.5M_2 + G_1 + 0.5G_2 \]  \hspace{1cm} (3)
\[ \pi_2 = B_2 + M_2 - 0.5M_1 + G_2 + 0.5G_1 \]  \hspace{1cm} (4)
\[ s_1 = G_1 - T_1 \]  \hspace{1cm} (5)
\[ s_2 = G_2 - T_2 \]  \hspace{1cm} (6)

The policy makers are the European central bank, the American central bank, the European government, and the American government. The targets of policy cooperation are zero inflation in Europe, zero inflation in America, zero unemployment in Europe, zero unemployment in America, a zero structural deficit in Europe, and a zero structural deficit in America. The instruments of policy cooperation are European money supply, American money supply, European government purchases, and American government purchases. There are six targets but only four instruments, so what is needed is a loss function.

We assume that the policy makers agree on a common loss function:

\[ L = \pi_1^2 + \pi_2^2 + u_1^2 + u_2^2 + s_1^2 + s_2^2 \]  \hspace{1cm} (7)

L is the loss caused by inflation, unemployment, and the structural deficit in each of the regions. We assume equal weights in the loss function. The specific target of policy cooperation is to minimize the loss, given the inflation functions, the
unemployment functions, and the structural deficit functions. Taking account of
equations (1) to (6), the loss function under policy cooperation can be written as
follows:

\[
L = (B_1 + M_1 - 0.5M_2 + G_1 + 0.5G_2)^2 \\
+ (B_2 + M_2 - 0.5M_1 + G_2 + 0.5G_1)^2 \\
+ (A_1 - M_1 + 0.5M_2 - G_1 - 0.5G_2)^2 \\
+ (A_2 - M_2 + 0.5M_1 - G_2 - 0.5G_1)^2 \\
+ (G_1 - T_1)^2 + (G_2 - T_2)^2
\]  

(8)

Then the first-order conditions for a minimum loss are:

\[
5M_1 = 2A_1 - A_2 - 2B_1 + B_2 - 3G_1 + 4M_2 \\
5M_2 = 2A_2 - A_1 - 2B_2 + B_1 - 3G_2 + 4M_1 \\
7G_1 = 2A_1 + A_2 - 2B_1 - B_2 + 2T_1 - 3M_1 - 4G_2 \\
7G_2 = 2A_2 + A_1 - 2B_2 - B_1 + 2T_2 - 3M_2 - 4G_1
\]  

(9) \hspace{1cm} (10) \hspace{1cm} (11) \hspace{1cm} (12)

Equation (9) shows the first-order condition with respect to European money
supply. Equation (10) shows the first-order condition with respect to American
money supply. Equation (11) shows the first-order condition with respect to
European government purchases. And equation (12) shows the first-order
condition with respect to American government purchases.

The cooperative equilibrium is determined by the first-order conditions for a
minimum loss. We assume \( T = T_1 = T_2 \). The solution to this problem is as
follows:

\[
3M_1 = 2A_1 + A_2 - 2B_1 - B_2 - 9T \\
3M_2 = 2A_2 + A_1 - 2B_2 - B_1 - 9T \\
G_1 = T \\
G_2 = T
\]  

(13) \hspace{1cm} (14) \hspace{1cm} (15) \hspace{1cm} (16)