Chapter 4
Monetary and Fiscal Cooperation

1. The Model

An increase in money supply lowers unemployment. On the other hand, it raises inflation. Correspondingly, an increase in government purchases lowers unemployment. On the other hand, it raises inflation. The policy makers are the central bank and the government. The targets of policy cooperation are zero inflation and zero unemployment.

The model of unemployment and inflation can be characterized by a system of two equations:

\[ u = A - \alpha M - \beta G \]  
\[ \pi = B + \alpha \varepsilon M + \beta \varepsilon G \]  

Of course this is a reduced form. Here \( u \) denotes the rate of unemployment, \( \pi \) is the rate of inflation, \( M \) is money supply, \( G \) is government purchases, \( \alpha \) is the monetary policy multiplier with respect to unemployment, \( \alpha \varepsilon \) is the monetary policy multiplier with respect to inflation, \( \beta \) is the fiscal policy multiplier with respect to unemployment, \( \beta \varepsilon \) is the fiscal policy multiplier with respect to inflation, \( A \) is some other factors bearing on the rate of unemployment, and \( B \) is some other factors bearing on the rate of inflation. The endogenous variables are the rate of unemployment and the rate of inflation.

According to equation (1), the rate of unemployment is a positive function of \( A \), a negative function of money supply, and a negative function of government purchases. According to equation (2), the rate of inflation is a positive function of \( B \), a positive function of money supply, and a positive function of government purchases. A unit increase in \( A \) raises the rate of unemployment by 1 percentage point. A unit increase in \( B \) raises the rate of inflation by 1 percentage point. A unit increase in money supply lowers the rate of unemployment by \( \alpha \) percentage
points. On the other hand, it raises the rate of inflation by $\alpha \varepsilon$ percentage points. A unit increase in government purchases lowers the rate of unemployment by $\beta$ percentage points. On the other hand, it raises the rate of inflation by $\beta \varepsilon$ percentage points.

The policy makers are the central bank and the government. The targets of policy cooperation are zero inflation and zero unemployment. The instruments of policy cooperation are money supply and government purchases. Thus there are two targets and two instruments. We assume that the policy makers agree on a common loss function:

$$L = \pi^2 + u^2$$  \hspace{1cm} (3)

$L$ is the loss caused by inflation and unemployment. For ease of exposition we assume equal weights in the loss function. The specific target of policy cooperation is to minimize the loss, given the inflation function and the unemployment function. Taking account of equations (1) and (2), the loss function under policy cooperation can be written as follows:

$$L = (B + \alpha \varepsilon M + \beta \varepsilon G)^2 + (A - \alpha M - \beta G)^2$$  \hspace{1cm} (4)

Then the first-order conditions for a minimum loss are:

$$\begin{align*}
(1 + \varepsilon^2)\alpha M &= A - \varepsilon B - (1 + \varepsilon^2)\beta G \\
(1 + \varepsilon^2)\beta G &= A - \varepsilon B - (1 + \varepsilon^2)\alpha M
\end{align*}$$  \hspace{1cm} (5) \hspace{1cm} (6)

Equation (5) shows the first-order condition with respect to money supply. And equation (6) shows the first-order condition with respect to government purchases. Obviously, equations (5) and (6) are identical. There are two endogenous variables, money supply and government purchases. On the other hand, there is only one independent equation. Thus there is an infinite number of solutions.

The cooperative equilibrium is determined by the first-order conditions for a minimum loss. The solution to this problem is as follows: