1. The Model

An increase in money supply lowers unemployment. On the other hand, it raises inflation. However, it has no effect on the structural deficit. Correspondingly, an increase in government purchases lowers unemployment. On the other hand, it raises inflation. And what is more, it raises the structural deficit. The target of the central bank is zero inflation. By contrast, the targets of the government are zero unemployment and a zero structural deficit.

The model of unemployment, inflation, and the structural deficit can be characterized by a system of three equations:

\[ u = A - M - G \]  \hspace{1cm} (1)
\[ \pi = B + M + G \]  \hspace{1cm} (2)
\[ s = G - T \]  \hspace{1cm} (3)

Here \( u \) denotes the rate of unemployment, \( \pi \) is the rate of inflation, \( s \) is the structural deficit ratio, \( M \) is money supply, \( G \) is government purchases, \( T \) is tax revenue at full-employment output, \( G - T \) is the structural deficit, \( A \) is some other factors bearing on the rate of unemployment, and \( B \) is some other factors bearing on the rate of inflation. The endogenous variables are the rate of unemployment, the rate of inflation, and the structural deficit ratio.

According to equation (1), the rate of unemployment is a positive function of \( A \), a negative function of money supply, and a negative function of government purchases. According to equation (2), the rate of inflation is a positive function of \( B \), a positive function of money supply, and a positive function of government purchases. According to equation (3), the structural deficit ratio is a positive function of government purchases. A unit increase in money supply lowers the rate of unemployment by 1 percentage point. On the other hand, it raises the rate
of inflation by 1 percentage point. However, it has no effect on the structural deficit ratio. A unit increase in government purchases lowers the rate of unemployment by 1 percentage point. On the other hand, it raises the rate of inflation by 1 percentage point. And what is more, it raises the structural deficit ratio by 1 percentage point.

The target of the central bank is zero inflation. The instrument of the central bank is money supply. By equation (2), the reaction function of the central bank is:

\[ M = -B - G \]  \hspace{1cm} (4)

Suppose the government raises government purchases. Then, as a response, the central bank lowers money supply.

The targets of the government are zero unemployment and a zero structural deficit. The instrument of the government is government purchases. There are two targets but only one instrument, so what is needed is a loss function. We assume that the government has a quadratic loss function:

\[ L_2 = u^2 + s^2 \]  \hspace{1cm} (5)

\( L_2 \) is the loss to the government caused by unemployment and the structural deficit. We assume equal weights in the loss function. The specific target of the government is to minimize the loss, given the unemployment function and the structural deficit function. Taking account of equations (1) and (3), the loss function of the government can be written as follows:

\[ L_2 = (A - M - G)^2 + (G - T)^2 \]  \hspace{1cm} (6)

Then the first-order condition for a minimum loss gives the reaction function of the government:

\[ 2G = A + T - M \]  \hspace{1cm} (7)

Suppose the central bank lowers money supply. Then, as a response, the government raises government purchases.