Plasmas belong to two different categories, unmagnetized and magnetized. The plasma in a fluorescent tube is unmagnetized, because the motion of electrons and ions is determined by electric fields and collisions, and the Earth magnetic field is too weak to bend the trajectories. The ionosphere, the magnetosphere, the solar wind, the interstellar medium and the solar surface are examples for natural magnetized plasmas. There, the motion of the particles is strongly affected by the magnetic field.

This chapter is focused on the motion of individual charged particles in given electric and magnetic fields. Of particular importance is the quest for magnetic confinement of plasmas. The inhomogeneity and curvature of magnetic field lines, or the variation of the fields in time cause complex particle motion. The model of single particle motion neglects the influence of particle currents on the electric and magnetic fields. In this respect, the model is still incomplete. Nevertheless, from an understanding of particle motion the reader will gain insight into the basic properties of a plasma that is subjected to electromagnetic fields.

3.1 Motion in Static Electric and Magnetic Fields

3.1.1 Basic Equations

The starting point for establishing the single-particle model is Newton’s equation\(^1\) for the motion of a particle of mass \(m\) and charge \(q\) in a given electric field \(\mathbf{E}\) and magnetic field \(\mathbf{B}\)

\[
m\mathbf{v} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) ,
\]

\(^1\) Sometimes called Newton-Lorentz equation
in which the dot represents the time derivative at the position of the particle. This equation can be solved in rigid mathematical terms only for simple cases, e.g., homogeneous and stationary fields.

### 3.1.2 Cyclotron Frequencies

Let us first consider the case of a homogeneous and stationary magnetic field \( \mathbf{B} = (0, 0, B_z) \) and a vanishing electric field \( \mathbf{E} = 0 \). The magnetic field is chosen as \( z \)-axis because of the cylindrical symmetry about the \( B \)-field direction. Then, we obtain Newton’s equation of motion in cartesian coordinates as

\[
\begin{align*}
\dot{v}_x &= +v_y \frac{q}{m} B_z \\
\dot{v}_y &= -v_x \frac{q}{m} B_z \\
\dot{v}_z &= 0.
\end{align*}
\]

(3.2)

By combining the equations for the \( x \) and \( y \)-motion we obtain the differential equation for a harmonic oscillator

\[
\ddot{v}_{x,y} = - \left( \frac{q B_z}{m} \right)^2 v_{x,y}.
\]

(3.3)

This harmonic oscillator describes a periodic motion at a frequency

\[
\omega_c = \frac{|q|}{m} B_z,
\]

(3.4)

which we call the cyclotron frequency. Inserting numbers for \( q, B, \) and \( m \) we find the cyclotron frequency of an electron in a magnetic field of 1 T at \( \omega_{ce} = 1.759 \times 10^{11} \text{ s}^{-1} = 2\pi \times 27.99 \text{ GHz} \). At the same magnetic field, the proton cyclotron frequency is \( \omega_{cp} = 9.579 \times 10^7 \text{ s}^{-1} = 2\pi \times 15.25 \text{ MHz} \).

In the \( x \)-\( y \) plane, a particle with perpendicular velocity \( v_\perp \) performs a circular orbit with the gyroradius or Larmor radius, named after the Irish physicist Joseph Larmor (1857–1942),

\[
r_L = \frac{v_\perp}{\omega_c}.
\]

(3.5)

When the initial velocity \( v_z \) along the magnetic field is nonzero, the orbit becomes a helix of constant pitch about the magnetic field direction \( (z) \). The motion about a magnetic field line is referred to as gyromotion or gyroorbit.

The sense of rotation about the magnetic field depends on the sign of the particle’s charge. Electrons move in a right-handed, positive ions in a left-handed orbit (see Fig. 3.1).