CENTRO INTERNAZIONALE MATEMATICO ESTIVO
(C.I.M.E.)

J. GUCKENHEIMER

SINGULARITIES IN SOAP-BUBBLE-LIKE AND SOAP-FILM-LIKE SURFACES

Corso tenuto a Varenna dal 24 agosto al 2 settembre 1972
Four fundamental questions in the calculus of variations are:

(1) The existence of solutions;
(2) The regularity of these solutions;
(3) The structure of the singular sets, if any, in these solutions; and
(4) The computation of the solutions.

For many geometric integrands with natural constraints, the first two problems are now solved. In particular, the two dimensional surfaces in $\mathbb{R}^3$ which are "soap-bubble-like" and "soap-film-like" (to be defined precisely below), have recently been shown by Almgren [AF1] to exist and be analytic submanifolds of $\mathbb{R}^3$ except for a compact singular set of zero two dimensional Hausdorff measure $^{(2)}$. In these notes I will give a complete answer to the third question above for these surfaces - that is, I will give a complete classification of the local structure of the singularities in "soap-bubble-like" and "soap-film-like" surfaces. In so doing, I will also introduce several techniques of geometric measure theory not presented elsewhere at this conference, such as tangent cones, "excess", and proof by contradiction; additionally, I will show how concepts presented elsewhere such as regarding

1. The research work was supported in part by a National Science Foundation Graduate Fellowship.

2. Hausdorff $k$-dimensional measure, denoted $\mathcal{H}^k$, agrees with any other reasonable definition of area on smooth $k$-dimensional submanifolds but additionally gives a precise meaning to area when singularities may be present; for an exact definition of Hausdorff area, see Almgren's lecture notes for this conference [AF2].