WAVE PROPAGATION AND HEAT CONDUCTION
IN A RANDOM MEDIUM

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Introduction

We shall give a fairly self-contained account of some results on waves in random media and related problems that we have considered in the past few years [1]-[6]. These results rely upon properties of solutions of differential equations with random coefficients, i.e., stochastic equations. We restrict attention to one-dimensional problems so that we are dealing with stochastic ordinary differential equations. There are a few results, at present, dealing with multidimensional problems at [cf. 12] but we shall not discuss these here.

1. FORMULATION OF THE PROBLEMS

We consider a one-dimensional medium occupying the interval [0, L] with a wave of unit amplitude incident from \( x < 0 \). Let \( u(x) \exp(-i\omega t) \) denote the complex-valued wave field at location \( x \) at time \( t \). The time factor will be omitted as is customary. The field \( u(x) \) satisfies the one-dimensional reduced wave equation

\[
\frac{d^2 u(x)}{dx^2} + k^2 n^2(x) u(x) = 0, \quad 0 < x < L.
\]

Here \( n(x) \) is the index of refraction, \( k = \omega/c \) is the wave number and \( c \) is the free space propagation speed. The index of refraction \( n(x) \) is a random process with known statistical properties to be described below.