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MARTINGALE THEORY - POTENTIAL THEORY

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Let \((\Omega, F, P)\) be a probability measure space and let \((X, G)\) be a measurable space.

A measurable function from the first space into the second is called a "random variable". The second space is called the "state space".

Let \(I\) be a totally ordered set. For each \(t\) in \(I\) let \(F_t\) be a sub \(\sigma\)-algebra of \(F\).

Let the state space be the line with its Borel sets and for each \(t\) in \(I\) let \(x(t, \cdot)\) be a random variable. Suppose that \(F_t\) increases with \(t\), that \(x(t, \cdot)\) is integrable and that \(s < t\) implies that

\[
\int_A x(t, \cdot) \, dP \leq \int_A x(s, \cdot) \, dP
\]

for every set \(A\) in \(F_s\). Then \(\{x(t, \cdot), t \in I\}\) is called a "supermartingale" (relative to the specified \(\sigma\)-algebras), a martingale if there is equality in (1).

In every version of potential theory there are analogues of superharmonic and harmonic functions.

The probabilistic analogues of these functions are supermartingales and martingales.

The notion of reduced function in this context becomes that of a reduced stochastic process and the usual arguments about reduced functions have corresponding versions.

Thus there is a completely probabilistic version of potential theory.

Besides this there is a combined version, Hunt potential theory, in which a Green kernel is defined probabilistically and both probabilistic and nonprobabilistic analysis are used.