CENTRO INTERNAZIONALE MATEMATICO ESTIVO

(C.I.M.E.)

METHODS OF ALGEBRAIC GEOMETRY IN CHAR. p
AND THEIR APPLICATIONS

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I. The aim of these lectures is to illustrate some of the algebraic
techniques needed for algebraic geometry in char. p, with a particular view
to the theory of algebraic surfaces and Enriques' classification. We shall
study the new char. p features of Kodaira's vanishing theorem, the complete-
ness of the characteristic system and the theory of the Picard variety,
together with the study of elliptic and quasi-elliptic fibrations, the study
of Enriques' surfaces in char. 2 and the characterization of abelian surfaces
by means of their numerical invariants.

Our basic list of notations will be as follows:

- $X$ = an algebraic variety
- $\text{Alb } X$ = Albanese variety of $X$
- $\text{Pic } X$ = Picard scheme of $X$
- $\text{Pic}^0 X$ = connected component of $0 \in \text{Pic } X$
- $q = \dim \text{Pic } X = \dim \text{Alb } X$, the irregularity of $X$
- $K = K_X$ = the canonical class of $X$
- $B_i = i$th Betti number
- $h^{p,q} = \dim H^q(X,\mathcal{O}_X^p)$
- $p_g = h^{0,2} = h^{2,0}$ if $\dim X = 2$
- $\omega_X = \Omega^2_X$ if $\dim X = 2$ and $X$ is smooth = in general, the
dualizing sheaf for Gorenstein varieties
- $h^q(F) = \dim H^q(X,F)$