ALGEBRAIC SURFACES WITH $q = p_g = 0$.

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CONTENTS

Introduction

Chapter I. Classical examples
   §1. The Enriques surface
   §2. The Godeaux surface
   §3. The Campedelli surface

Chapter II. Elliptic surfaces
   §1. Generalities
   §2. Torsion
   §3. The fundamental group

Chapter III. Surfaces of general type
   §1. Some useful lemmas
   §2. Numerical Godeaux surfaces
   §3. Numerical Campedelli surfaces
   §4. Burniat's examples
   §5. Surfaces with $p^{(1)} = 9$
   §6. Concluding remarks

Bibliography

Epilogue
1. **Notations.** Let $F$ be a complex algebraic surface. We will use the following standard notations:

- $\mathcal{O}_F$: the structure sheaf of $F$.
- $\mathcal{O}_F(D)$: the invertible sheaf associated with a divisor $D$ on $F$.
- $K_F = -c_1(F)$: minus the first Chern class of $F$ or a canonical divisor on $F$.
- $\omega_F = \mathcal{O}_F(K_F)$: the canonical sheaf of $F$.
- $h^1(D)$: the dimension of the space $H^1(F, \mathcal{O}_F(D))$.
- $p_g(F) = h^0(K_F) = h^2(\mathcal{O}_F)$: the geometric genus of $F$.
- $q(F) = h^1(K_F) = h^1(\mathcal{O}_F)$: the irregularity of $F$.
- $K_F^2$: the self-intersection index of $K_F$.
- $p^{(1)}(F) = K_F^2 + 1$, where $F'$ is a minimal model of a non-rational surface $F$; the linear genus of $F$.
- $c_2(F)$: the topological Euler-Poincare characteristic of $F$.
- $p_{ar}(F) = -q(F) + p_g(F) = 1/12(K_F^2 + c_2(F)) - 1$: the arithmetical genus.
- $p_n(F) = h^0(nK_F)$: the $n$-genus of $F$. 

*Introduction*