SOME REMARKS ON THE CRITICAL PROBLEM

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Andrea Brini
(Università di Bologna)

Introduction

In 1970, Crapo and Rota proposed a reformulation of a classical extremal problem on finite vector spaces, namely, the problem of finding the largest dimension of a subspace having empty intersection with a given set of vectors $S$. Besides achieving an higher degree of generality, their most pleasing result was that of showing this problem to be equivalent to a problem concerning the location of zeroes of the characteristic polynomial associated to the matroid structure induced on the set $S$.

Recently, Kung, Murty and Rota succeeded in proving that an analogous problem on finite abelian groups admits a solution in terms of the location of zeroes of another function, the so-called Rèdei zeta function of a set of points in a Dirichlet lattice.

In this paper, we deal with an unified exposition of some extremal problems which can be solved in this way. In section 1, we describe the critical problem for finite abelian groups and its connection with a class of Rèdei zeta functions. In section 2, we recall Crapo and Rota's Theorem relating to the critical problem for finite vector spaces, here seen as a simple consequence of the preceding result on groups. Section 3 summarizes specializations to graph colourings. In section 4, we recall connections with the (linear) coding problem; in particular, we show that some classical results on bounds can be easily derived from purely matroid theoretic facts about Hamming spheres.