Chapter 11
Spherical Microphone Array Beamforming

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Abstract Spherical microphone arrays have been recently studied for spatial sound recording, speech communication, and sound field analysis for room acoustics and noise control. Complementary studies presented progress in beamforming methods. This chapter reviews beamforming methods recently developed for spherical arrays, from the widely used delay-and-sum and Dolph-Chebyshev, to the more advanced optimal methods, typically performed in the spherical harmonics domain.

11.1 Introduction

The growing interest in spherical microphone arrays can probably be attributed to the ability of such arrays to measure and analyze three-dimensional sound fields in an effective manner. The other strong point of these arrays is the ease of array processing performed in the spherical harmonics domain. The papers by Meyer and Elko [1] and Abhayapala and Ward [2] presented the use of spherical harmonics in spherical array processing and inspired a growing research activity in this field. The studies that followed provided further insight into spherical arrays from both theoretical and experimental view points.

This chapter presents an overview of some recent results in one important aspect of spherical microphone arrays, namely beamforming. The wide range of beamforming methods available in the standard array literature have been recently adapted for spherical arrays, typically formulated in the spherical harmonics domain, facilitating spatial filtering in three-dimensional sound fields. The chapter starts with an overview of spherical array processing,
presenting the spherical Fourier transform and array processing in the space and transform domains. Then, standard beam pattern design methods such as regular, delay-and-sum, and Dolph-Chebyshev are developed for spherical arrays. Optimal and source-dependent methods such as minimum variance and null-steering based methods are presented next. The chapter concludes with more specific beamforming techniques, such as steering beam patterns of arbitrary shapes, beamforming for sources in the near-field of the array, and direction-of-arrival estimation.

11.2 Spherical Array Processing

The theory of spherical array processing is briefly outlined in this section. Consider a sound field with pressure denoted by $p(k, r, \Omega)$, where $k$ is the wave number, and $(r, \Omega) \equiv (r, \theta, \phi)$ is the spatial location in spherical coordinates [3]. The spherical Fourier transform of the pressure is given by [3]

$$p_{nm}(k, r) = \int_{\Omega \in S^2} p(k, r, \Omega) Y_n^m(\Omega) d\Omega,$$

with the inverse transform relation:

$$p(k, r, \Omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} p_{nm}(k, r) Y_n^m(\Omega),$$

where $\int_{\Omega \in S^2} d\Omega \equiv \int_0^{2\pi} \int_0^{\pi} \sin \theta d\theta d\phi$ and $Y_n^m(\Omega)$ is the spherical harmonics of order $n$ and degree $m$ [3]:

$$Y_n^m(\Omega) \equiv \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}} P_n^m(\cos \theta) e^{im\phi},$$

where $i = \sqrt{-1}$ and $P_n^m(\cdot)$ is the associated Legendre function. A sound field composed of a single plane wave is of great importance for beamforming because beam patterns are typically measured as the array response to a single plane wave [4]. Therefore, we consider a sound field composed of a single plane wave of amplitude $a(k)$, with an arrival direction $\Omega_0$, in which case $p_{nm}$ can be written as [5]

$$p_{nm}(k, r) = a(k) b_n(kr) Y_n^m(\Omega_0),$$

where $b_n(kr)$ depends on the sphere boundary and has been presented for rigid sphere, open sphere: