Financial Time Series Models

More haste, less speed.

This section deals with financial time series analysis. The statistical properties of asset and return time series are influenced by the media (daily news on the radio, television and newspapers) that informs us about the latest changes in stock prices, interest rates and exchange rates. This information is also available to traders who deal with immanent risk in security prices. It is therefore interesting to understand the behavior of asset prices. Economic models on the pricing of securities are mostly based on theoretical concepts which involve the formation of expectations, utility functions and risk preferences. In this section we concentrate on answering the empirical questions. Firstly, given a data set we aim to specify an appropriate model reflecting the main characteristics of the empirically observable stock price process and we wish to know whether the assumptions underlying the model are fulfilled in reality or whether the model has to be modified. A new model on the stock price process could possibly effect the function of the markets. To this end we apply statistical tools to empirical data and start with considering the concepts of univariate analysis before moving on to multivariate time series.

Exercise 11.1. Let $X$ be a random variable with $E(X^2) < \infty$ and define a stochastic process

$$X_t \overset{\text{def}}{=} (-1)^t X, \ t = 1, 2, \ldots$$

(11.1)

(a) What do the paths of this process look like?

(b) Find a necessary and sufficient condition for $X$ such that the process $\{X_t\}$ is strictly stationary.

(c) Find a necessary and sufficient condition for $X$ such that $\{X_t\}$ is covariance (weakly) stationary.

(d) Let $X$ be such that $\{X_t\}$ is covariance (weakly) stationary. Calculate the autocorrelation $\rho_{\tau}$. 
(a) If for example $X(\omega) = 0.5836$, then the corresponding sample path is given in the Figure 11.1.

\[\begin{align*}
\text{Sample Path} & \\
0 & \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \\
-1 & \quad -0.8 \quad -0.6 \quad -0.4 \quad -0.2 \quad 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1
\end{align*}\]

Fig. 11.1. Sample path for the case $X(\omega) = 0.5836$.

(b) According to the definition, the stochastic process $X_t$ is strictly stationary if for any $t_1, \ldots, t_n$ and for all $n, s \in \mathbb{Z}$ it holds that

\[P(X_{t_1} \leq x_1, X_{t_2} \leq x_2, \ldots, X_{t_n} \leq x_n) = P(X_{t_1+s} \leq x_1, X_{t_2+s} \leq x_2, \ldots, X_{t_n+s} \leq x_n).\]

In our special case of the process \{${X_t}$\} defined by (11.1), the definition of strict stationarity reduces to

\[P(X_1 \leq a, X_2 \leq b) = P(X_2 \leq a, X_3 \leq b).\]

We check that this condition is fulfilled if, and only if, the distribution of $X$ is symmetric, i.e. $P(X \leq x) = P(-X \leq x)$ for all $x$.

If the distribution of $X$ is symmetric, it holds:

\[
\begin{align*}
P(X_1 \leq a, X_2 \leq b) &= P(-X \leq a, X \leq b) \\
&= P(-b \leq -X \leq a) \\
&= P(-X \leq a) - P(-X < -b).
\end{align*}
\]

Because of the symmetry of the distribution of $X$, we obtain:

\[
\begin{align*}
P(-X \leq a) - P(-X < -b) &= P(X \leq a) - P(X < -b) \\
&= P(-b \leq X \leq a) \\
&= P(X \leq a, -X \leq b) \\
&= P(X_2 \leq a, X_3 \leq b)
\end{align*}
\]