Time Series with Stochastic Volatility

Try to steal a chicken, but end up with losing the rice.

In the previous chapters, we have already discussed that volatility plays an important role in modeling financial systems and time series. Unlike the term structure, volatility is unobservable and thus must be estimated from the data. Reliable estimations and forecasts of volatility are important for large credit institutes where volatility is directly used to measure risk. The risk premium, for example, is often specified as a function of volatility. It is interesting to find an appropriate model for volatility. The capability of macroeconomic factors to forecast volatility has already been examined in the literature. Although macroeconomic factors have some forecasting capabilities, the most important factor seems to be the lagged endogenous return. As a result recent studies are mainly concentrated on time series models.

Stock, exchange rates, interest rates and other financial time series have stylized facts that are different from other time series. A good candidate for modeling financial time series should represent the properties of stochastic processes. Neither the classic linear AR or ARMA processes nor the nonlinear generalizations can fulfill this task. In this chapter we will describe the most popular volatility class of models: the ARCH (autoregressive conditional heteroscedasticity) model that can replicate these stylized facts appropriately.

Exercise 13.1. For the time series of daily DAX and FTSE 100 returns from 1 January 1998 to 31 December 2007, graphically illustrate the following empirical functions:

a) autocorrelation function for plain returns,

b) partial autocorrelation function for plain returns,

c) autocorrelation function for squared returns, and

d) autocorrelation function for absolute returns.
In addition, compute the Ljung-Box \( (Q_m^*) \) test statistics, for plain returns, squared returns and absolute returns, as well as the ARCH test statistics for plain returns. Select the number of lags \( m \) close to \( \log(n) \), where \( n \) denotes the sample size, see (Tsay 2002).

Are the DAX and FTSE 100 return processes in the period under review:

a) stationary,

b) serially uncorrelated,

c) independent?

Select an appropriate linear time series model for the return processes. Are ARCH and GARCH models appropriate for modeling the volatility processes of the analyzed returns?

The graphical illustration of the empirical autocorrelation functions and the partial correlation functions for analyzed time series are given in Figures 13.1 and 13.2 for the DAX index and the FTSE 100 index, respectively. In the period under review, there are \( n = 2807 \) observed returns. By selecting \( m = 8 \), the computed values for the Ljung-Box Portmanteau statistics and the ARCH test statistics are given in Table 13.1.

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**Fig. 13.1.** The autocorrelation function and the partial autocorrelation function plots for DAX plain, squared and absolute returns, from 1 January 1998 to 31 December 2007. ☑ SFSgarch