Portfolio Credit Risk

Winning is earning, losing is learning.

Financial institutions are interested in loss protection and loan insurance. Thus determining the loss reserves needed to cover the risk stemming from credit portfolios is a major issue in banking. By charging risk premiums a bank can create a loss reserve account which it can exploit to be shielded against losses from defaulted debt. However, it is imperative that these premiums are appropriate to the issued loans and to the credit portfolio risk inherent to the bank. To determine the current risk exposure it is necessary that financial institutions can model the default probabilities for their portfolios of credit instruments appropriately. To begin with, these probabilities can be viewed as independent but it is apparent that it is plausible to drop this assumption and to model possible defaults as correlated events.

In this chapter we give examples of the different methods to calculate the risk exposure of possible defaults in credit portfolios. Starting with basic exercises to determine the loss given default and the default probabilities in portfolios with independent defaults, we move on to possibilities to model correlated defaults by means of the Bernoulli and Poisson mixture models.

Exercise 18.1 (Expected Loss). Assume a zero coupon bond repaying full par value 100 with probability 95% and paying 40 with probability 5% in one year. Calculate the expected loss.

Probability of default in this exercise is PD = 5%, exposure at default (EAD) is EAD = 100 and loss given default (LGD) is LGD = 60%. Hence, the expected loss is:

\[ E(L) = EAD \cdot LGD \cdot PD = 100 \times 0.6 \times 0.05 = 3 \]

Exercise 18.2. Consider a bond with the following amortization schedule: the bond pays 50 after half a year \((T_1)\) and 50 after a full year \((T_2)\). In case of default before \(T_1\) the bond pays 40 and in case of default in \([T_1, T_2]\) pays 20. Calculate the expected loss when the probabilities of default in \([0, T_1)\) and \([T_1, T_2]\) are

i) 1% and 4%

ii) 2.5% and 2.5%
iii) 4% and 1%
respectively.

Following the expected loss logic (Exercise 18.1) one obtains

\[ i) \quad E(\tilde{L}) = 60 \times 0.010 + 30 \times 0.040 = 0.6 + 1.20 = 1.80 \]
\[ ii) \quad E(\tilde{L}) = 60 \times 0.025 + 30 \times 0.025 = 1.5 + 0.75 = 2.25 \]
\[ iii) \quad E(\tilde{L}) = 60 \times 0.040 + 30 \times 0.010 = 2.4 + 0.30 = 2.70. \]

Note that the time of default has an impact on the expected loss. Front loaded default curves generate a larger expected loss than back loaded curves.

Exercise 18.3 (Joint Default). Consider a simplified portfolio of two zero coupon bonds with the same probability of default (PD), par value 1 and zero recovery. The loss events are correlated with correlation \( \rho \).

i) Calculate the loss distribution of the portfolio,

ii) Plot the loss distribution for PD = 20% and \( \rho = 0; 0.2; 0.5; 1. \)

i) Let \( L_1 \) and \( L_2 \) be the loss of the first and second bond respectively. Then

\[
\text{Corr}(L_1, L_2) = \frac{\text{Cov}(L_1, L_2)}{\sqrt{\text{Var}(L_1) \text{Var}(L_2)}} = \frac{E(L_1L_2) - E(L_1)E(L_2)}{\text{Var} L_1} = \frac{P(L_1 = 1, L_2 = 1) - PD^2}{(1 - PD)PD}
\]

and

\[ P(L_1 = 1, L_2 = 1) = \rho(1 - PD)PD + PD^2. \]

Note that for \( \rho = 0 \), i.e. the losses are uncorrelated, the joint probability is equal to \( PD^2 \). For \( \rho = 1 \) they are linearly dependent and the joint probability is equal to \( PD \).

\[ P(L_1 = 1, L_2 = 0) + P(L_1 = 1, L_2 = 1) = P(L_1 = 1) = PD \]

and hence

\[ P(L_1 = 1, L_2 = 0) = PD - \rho(1 - PD)PD - PD^2 = PD(1 - PD)(1 - \rho). \]