Chapter 22
Fractional Analysis of Traffic Dynamics

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Abstract This article presents a dynamical analysis of several traffic phenomena, applying a new modelling formalism based on the embedding of statistics and Laplace transform. The new dynamic description integrates the concepts of fractional calculus leading to a more natural treatment of the continuum of the Transfer Function parameters intrinsic in this system. The results using system theory tools point out that it is possible to study traffic systems, taking advantage of the knowledge gathered with automatic control algorithms.

22.1 Dynamical Analysis

In the dynamical analysis of Traffic can be applied the tools of systems theory. In this line of thought, a set of simulation experiments are developed in order to estimate the influence of the vehicle speed \( v(t;x) \), the road length \( l \) and the number of lanes \( n_l \) in the traffic flow \( \phi(t;x) \) at time \( t \) and road coordinate \( x \). For a road with \( n_l \) lanes the Transfer Function (TF) between the flow measured by two sensors is calculated by the expression:

\[
G_{r,k}(s;x_j,x_i) = \Phi_r(s;x_j)/\Phi_k(s;x_i)
\]  

(22.1)

where \( k, r = 1, 2, \ldots, n_l \) define the lane number and, \( x_i \) and \( x_j \) represent the road coordinates (\( 0 \leq x_i \leq x_j \leq l \)). The Fourier Transform for each traffic flow is:

\[
\Phi_r(s;x_j) = F\{\phi_r(t;x_j)\}
\]

\[
\Phi_k(s;x_i) = F\{\phi_k(t;x_i)\}
\]  

(22.2)

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It should be noted that the traffic flow is a time variant system but, in the sequel, it is shown that the Fourier transform can be used to analyse the system dynamics.

The first group of experiments considers a one-lane road (i.e., \( k = r = 1 \)) with length \( l = 1,000 \text{ m} \). Across the road are placed \( n_s \) sensors equally spaced. The first sensor is placed at the beginning of the road (i.e., at \( x_i = 0 \)) and the last sensor at the end (i.e., at \( x_j = l \)). Therefore, we calculate the \( TF \) between two traffic flows at the beginning and the end of the road such that, \( \phi_1(t;0) \in [0.12, 1] \text{ vehicles s}^{-1} \) for vehicle speed \( v_1(t;0) \in [30, 70] \text{ km h}^{-1} \) that is, for \( v_1(t;0) \in [v_{av} - \Delta v, v_{av} + \Delta v] \), where \( v_{av} = 50 \text{ km h}^{-1} \) is the average vehicle speed and \( \Delta v = 20 \text{ km h}^{-1} \) is the maximum speed variation. These values are generated according to a uniform probability distribution function.

The polar plot result for the \( TF \) between the traffic flow at the beginning and end of the one-lane road \( G_{1,1}(s; 1000, 0) = \Phi_1(s; 1000)/\Phi_1(s; 0) \) is distinct from those usual in systems theory revealing a large variability, as revealed by Fig. 22.1. Moreover, due to the stochastic nature of the phenomena involved different experiments using the same input range parameters result in different \( TFs \).

In fact traffic flow is a complex system but it was shown [1] that, by embedding statistics and Fourier transform (leading to the concept of Statistical Transfer Function (\( STF \)) [3], we could analyse the system dynamics in the perspective of systems theory. To illustrate the proposed modelling concept (\( STF \)), the simulation was repeated for a sample of \( n = 2,000 \) vehicles and it was observed the existence of a convergence of the \( STF \), \( T_{1,1}(s; 1,000, 0) \), as show in Fig. 22.2, for a one-lane road with length \( l = 1,000 \text{ m} \) \( \phi_1(t;0) \in [0.12, 1] \text{ vehicles s}^{-1} \) and \( v_1(t;0) \in [30, 70] \text{ km h}^{-1} \).

The chart has characteristics similar to those of a low-pass filter with time delay, common in systems involving transport phenomena. Nevertheless, in our case we

![Fig. 22.1 Polar diagram of TF for \( n = 1 \) experiment, with \( \phi_1(t;0) \in [0.12, 1] \text{ vehicles s}^{-1} \) and \( v_1(t;0) \in [30, 70] \text{ km h}^{-1} \left( v_{av} = 50 \text{ km h}^{-1}, \Delta v = 20 \text{ km h}^{-1}, l = 1,000 \text{ m and } n_l = 1 \right) \)