Relativistic High Order Particle Treatment for Electromagnetic Particle-In-Cell Simulations

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Abstract A recently developed high order field solver for the complete Maxwell equations provides all information needed by a new relativistic particle push method based on a truncated Taylor series expansion up to the desired order of convergence. The property and capability of this approach is demonstrated for different numerical experiments.

1 Introduction

The electrical behavior of technical systems like microwave devices is substantially influenced by a flow of charged particles forming a non-neutral plasma inside. A detailed understanding of the phenomena caused by this plasma requires the solution of the Maxwell-Vlasov equations for realistic configurations. An attractive numerical technique to do this is the Particle-in-Cell (PIC) method. In essence, the basic idea of the PIC approach can be summarized as follows: At each time step the electromagnetic fields are obtained by the numerical solution of the full set of the nonstationary Maxwell equations, where different kind of methods like finite volume [1] or discontinuous Galerkin schemes of free selectable order of convergence are applied. Note that the Maxwell part of this solver comprises additionally a purely hyperbolic divergence correction mechanism [2] to ensure the constrain of charge conservation during the simulation. Subsequently, these fields are interpolated to the actual locations of the charged plasma particles which are then pushed by the Lorentz force and redistributed in phase space according to the usual laws of
dynamics. Afterwards, the particles have to be located with respect to the computational grid in order to assign the contribution of each charge to the changed charge and current density to the nodes of the mesh. These densities are the sources for the Maxwell equations of the subsequent iteration cycle which finally guarantee a self-consistent computation of the interaction of the electromagnetic fields with the charged plasma particles. The recent development of high order Maxwell solvers for electromagnetic wave propagation offers the possibility to construct high order PIC algorithms for the numerical solution of the Maxwell-Vlasov equations. In this context the central challenge is the high order computation of the phase space coordinates of the plasma particles. In the present paper we introduce a new particle treatment based on a Taylor series expansion (TSE) of the phase space variables in time up to the selected order of accuracy of the field solver. The important requirement to establish this high order phase space coordinates calculation is that all necessary spatial as well as temporal derivatives of the electromagnetic fields for each particle are known from the Maxwell solver. This knowledge of the high order derivatives is extensively used to obtain the TSE of the phase space coordinates of the charge as it is explained below.

In the next section the formulation of the governing equations is given and the numerical approximation is discussed. In section 3 we demonstrate the capability and reliability of the high order particle (HIOP) procedure by means of different numerical simulation experiment. Finally, a conclusion and a short outlook is given in section 4.

2 Governing Equations and Numerical Approximation

2.1 Equation of Motion for Charged Particles

The general solution of the Vlasov equation is given by its characteristics

\[
\frac{d}{dt} (m U) = F_L(v, x, t), \quad \frac{dx}{dt} = v
\]  

with the Lorentz force

\[
F_L = q \left[ E(x, t) + v \times B(x, t) \right]
\]

acting on charge \( q \) with mass \( m \), where \( E \) and \( B \) denote the external applied or/and self electric field and magnetic induction, respectively. The velocity of the charged particle \( v \) is related to the space component of the 4-velocity \( U \) according to [3]

\[
v = \hat{\gamma} \ U(t), \quad \hat{\gamma}(U) = \left( 1 + \frac{U \cdot U}{c^2} \right)^{-1/2}
\]

with the inverse relativistic factor \( \hat{\gamma} \), where \( c \) is the speed of light. As a consequence of the latter relation, the phase space coordinates \((v, x)\) may be regarded