10  Statistics on Directional Data

10.1  Introduction

Methods for analyzing circular and spherical data are widely used in earth sciences. For instance, structural geologists measure and analyze the orientation of slickensides (or striae) on fault planes. Circular statistics is also common in paleomagnetic applications. Microstructural investigations include the analysis of grain shapes and quartz c-axis orientations in thin sections. Paleoenvironmentalists also reconstruct paleocurrent directions from fossil alignments (Fig. 10.1). In principle, two types of directional data exist in earth sciences: directional data *sensu stricto*, and oriented data. Directional data have a true polarity, such as the paleocurrent direction of a river as documented by flute marks, or the flow direction of a glacier as indicated by glacial striae. Oriented data describe axial data and lines without any sense of direction, such as the orientation of joints.

MATLAB is not the first choice for analyzing directional data since it does not provide the relevant functions, such as an algorithm to compute the probability distribution function of a von Mises distribution or to run a Rayleigh’s test for the significance of a mean direction. Earth scientists have therefore developed numerous stand-alone programs with which to analyze such data, e.g., the excellent software developed by Rick Allmendinger, available for Mac OS 9 and OS X as well as for Microsoft Windows:

http://www.geo.cornell.edu/geology/faculty/RWA/programs.html

The following tutorial on the analysis of directional data is independent of these tools. It provides simple MATLAB codes to display directional data, to compute the von Mises distribution and to run simple statistical tests. The first section introduces rose diagrams as the most widely used method to display directional data (Section 10.2). With a similar concept to Chapter 3 on univariate statistics, the next sections are on empirical and theoretical distributions to describe directional data (Sections 10.3 and 10.4). The last three sections then describe the three most important tests for directional data.
Orthoceras fossils from an outcrop Neptuni Acrar near Byxelkrok on Öland, Sweden. *Orthoceras* is a cephalopod with a straight shell and that lived in the Ordovician era, about 450 million years ago. Such elongated, asymmetric objects tend to orient themselves in the hydrodynamically most stable position. The fossils can therefore indicate paleocurrent directions. The statistical analysis of cephalopod orientations at Neptuni Acrar reveals a significant southerly paleocurrent direction, which is in agreement with the paleogeographic reconstructions.

data, these being the tests for randomness of directional data (Section 10.5), for the significance of a mean direction (Section 10.6), and for the difference between two sets of directional data (Section 10.7).

### 10.2 Graphical Representation

The classic way to display directional data is the rose diagram. A rose diagram is a histogram for measurements of angles. In contrast to a bar histogram with the height of the bars proportional to frequency, the rose diagram comprises segments of a circle with the radius of each sector being proportional to the frequency. We use synthetic data to illustrate two types of rose diagrams used to display directional data. We load a set of directional data from the file *directional_1.txt*. 